

# A New Reliability-Based Optimal Design Algorithm of Electromagnetic Problems with Uncertain Variables: Multi-objective Approach

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**Abstract** – For the optimal design of electromagnetic device involving uncertainties in design variables, this paper proposes a new reliability-based optimal design algorithm for multiple constraints problems. Through optimizing the nominal objective function and maximizing the minimum reliability, a set of global optimal reliable solutions representing different reliability levels are obtained by the multi-objective particle swarm optimization algorithm. Applying the sensitivity-assisted Monte Carlo simulation method, the numerical efficiency of optimization procedure is guaranteed. The proposed reliability-based algorithm supplying multi-reliable solutions is investigated through applications to analytic examples and the optimal design of two electromagnetic problems.

**Keywords:** Monte Carlo simulation, Multi-objective optimal design, Reliability calculation, Sensitivity analysis

## 1. Introduction

In order to cope with uncertain design variables (UDVs), numerous endeavors have been made in the electrical engineering to develop robust optimal design algorithms through minimizing performance variations, adopting the gradient index method and worst-case scenario approximation [1, 2]. When a system includes critical constraints with UDVs, however, it is more imperative to keep the probability of failure event for constraints less than a predefined value or ensure the reliability with respect to constraints to a certain confidence level.

Recently, researches on the reliability analysis [3] and reliability-based optimal design (RBOD) have been set out in electrical engineering [4] based on algorithms developed in mechanical engineering [5, 6]. These researches mainly pursue a constraint-reliable optimal design by means of reliability analysis such as reliability index approach (RIA) [5] and performance measure approach (PMA) [6].

As shown in Fig. 1, the deterministic/classical optimization finds the best performance solution disregarding uncertainty; the robust optimization seeks a solution giving minimum variation of performance within the uncertainty set. The RBOD, on the other hand, tries to select the best performance one only among solutions having higher level of reliability than a predefined value.

Most conventional RBODs in all published works have treated reliability as a probabilistic constraint [4-6]. In

detail, a constraint involving UDVs is firstly transformed into a reliability condition that an optimal solution should have higher reliability than a predefined value, and then an optimal design is found through optimization [7]. Although this method finds a reliable solution corresponding to the target reliability, it provides information only for the obtained optimal solution not for other solutions. However, if the reliability is treated as an additional independent objective function to be maximized, a set of Pareto-optimal solutions will be obtained in objective and constraint functions space, from which a designer may realize how the reliable solution changes with different level of reliability, and decide a suitable solution according to a particular application.

This paper, to seek for multiple reliable solutions, proposes a novel RBOD algorithm which treats probabilistic constraints as additional independent objective function to be maximized. In the algorithm, the reliability analysis is achieved by using sensitivity-assisted Monte Carlo simulation (SA-MCS) proposed in our previous research [8], and multi-guider and cross-searching multi-objective particle swarm optimization (MGC-PSO) is employed to get global Pareto-optimal design.

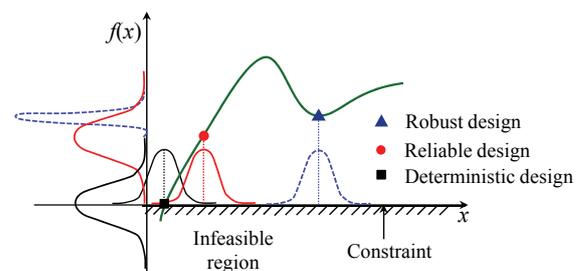


Fig. 1. Illustration of different optimal solutions for minimization problem

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## 2. Reliability Analysis Using Sensitivity Assisted Monte Carlo Simulation Method (SA-MCS)

In this paper, all design variables,  $\mathbf{x} \in R^n$ , are assumed to be uncertain ones and follow independent Gaussian distribution. At a specific design,  $\mathbf{x}_0$ , its reliability for a constraint,  $g(\mathbf{x}) \leq 0$ , is defined, by using Monte Carlo simulation (MCS), as the probability of satisfying the constraints as UDV's change around  $\mathbf{x}_0$  as follows [8, 9]:

$$R(g(\mathbf{x}_0) \leq 0) = \frac{1}{N} \sum_{i=1}^N I[g(\xi_i)], \quad \xi_i \in U(\mathbf{x}_0) \quad (1a)$$

$$I[g(\xi_i)] = \begin{cases} 1 & \text{if } g(\xi_i) \leq 0 \\ 0 & \text{otherwise} \end{cases} \quad (1b)$$

where  $N$  is the number of test perturbed designs, and the uncertainty set  $U(\mathbf{x}_0)$  is defined as:

$$U(\mathbf{x}_0) = \{ \xi \in R^n \mid \mathbf{x}_0 - k\boldsymbol{\sigma} \leq \xi \leq \mathbf{x}_0 + k\boldsymbol{\sigma} \} \quad (2)$$

where  $\boldsymbol{\sigma}$  is a vector of standard deviations and  $k$  is constant corresponding to required confidence level, i.e.,  $k=1.96$  when the confidence level is 95%.

In SA-MCS method, to save the computing time required in (1), the constraint function value at a perturbed design is approximated as follows:

$$g(\xi) \cong g(\mathbf{x}_0) + \nabla g(\mathbf{x}_0) \cdot (\xi - \mathbf{x}_0), \quad \xi \in U(\mathbf{x}_0) \quad (3)$$

If the constraint involves the finite element method (FEM) to be analyzed, the gradient vector can be obtained through design sensitivity analysis [10]. The computational cost required in reliability analysis for a constraint at a specific design is only one more additional FEM call to calculate the gradient vector.

## 3. Reliability-Based Optimal Design Algorithms

A typical optimization problem subject to  $m$  constraints is generally formulated as follows:

$$\begin{aligned} & \text{Minimize } f(\mathbf{x}) \\ & \text{subject to } g_i(\mathbf{x}) \leq 0, \quad i = 1, \dots, m \end{aligned} \quad (4)$$

In the deterministic optimization, an advanced global optimizer may be adopted to yield an optimal design with the best performance. This method does not pay regard to the uncertainties in design variables, thereby the optimal solution, in general, locates on or very close to the constraint boundary, as the design (A) shown in Fig. 2.

### 3.1 Conventional RBOD

The conventional RBOD introduces reliability concept

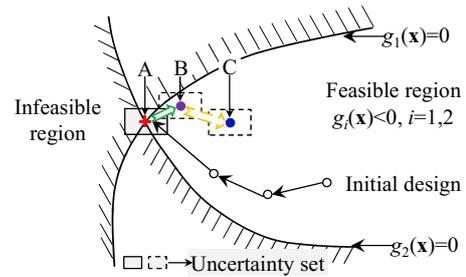


Fig. 2. Description of the reliability-based design optimization

to ensure that an optimal solution remains inside the feasible region even with perturbation, and is formulated as follows:

$$\begin{aligned} & \text{Minimize } f(\mathbf{x}) \\ & \text{subject to } R(g_i(\mathbf{x}) \leq 0) \geq R_{t,i}, \quad i = 1, \dots, m. \end{aligned} \quad (5)$$

where  $R_{t,i}$  is the target reliability for the  $i$ -th constraint.

Most conventional RBOD algorithms, as shown in Fig. 2, start with finding the deterministic solution (design A). Then the solution is moved back to the feasible region by finding a reliable optimum (design B or design C) to guarantee the target reliability. It is obvious, in this method, that only one solution is available corresponding to the target reliability. For a different level of reliability, this method needs to be run independently.

In order to find a global optimal solution, in this paper, (5) is solved by using particle swarm optimization (PSO).

### 3.2 Proposed RBOD-Multi-objective approach

The conventional RBODs seek for an optimal solution, which guarantees the target reliability as well as good performance. In the RBODs, furthermore, a trade-off between performance and reliability level is inevitable. From this viewpoint, the constraints in (4) can be treated as an additional independent objective function to be maximized. Therefore, the optimization problem (4) is formulated in a multi-objective way to get Pareto-optimal solutions as follows:

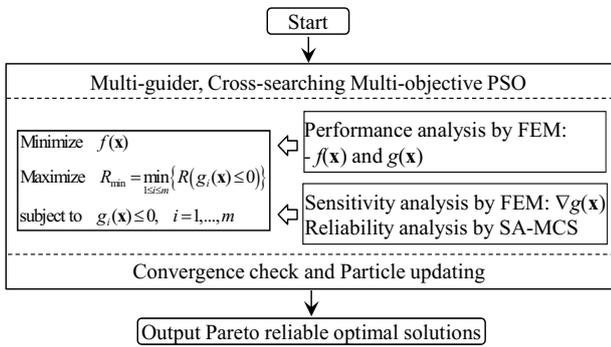
$$\begin{aligned} & \text{Minimize } f(\mathbf{x}) \\ & \text{Maximize } R_{\min} = \min_{1 \leq i \leq m} \{ R(g_i(\mathbf{x}) \leq 0) \} \end{aligned} \quad (6)$$

where  $R_{\min}$  is the minimum reliability among all constraints.

In the solution of (6), the MGC-MOPSO (a multi-objective version of PSO which utilizes multi-guiders and cross-searching strategy) is adopted [11]. We can get global Pareto-optimal solutions, which ranges from the design with the best performance and the lowest level of reliability to one with the worst performance and the highest level of reliability. Therefore, the designer can easily see how the reliable solutions change with different reliability levels.

**Table 1.** Comparison of reliability analysis and different RBDO algorithms

Methods	Treating reliability	Reliability calculation	Optimization method	Optimal solution	Ref
Conventional RBOD	constraint	RIA & PMA	Sequential quadratic programming	Single local optimal solution	[4], [6]
		PMA	Genetic algorithm	Single global optimal solution	[12]
Proposed RBOD	Objective function	SA-MCS	MGC-MOPSO	Pareto-optimal solutions	–



**Fig. 3.** Flowchart of the proposed RBOD algorithm

Due to adoption of the SA-MCS method, for a specified design, the computational cost required in performance and reliability analyses in (6), is just  $(1+m)$  times of FEM calls.

The numerical implementation of the proposed RBOD algorithm is shown in Fig. 3. The proposed RBOD algorithm is compared, in Table 1, with conventional ones from the viewpoint of reliability calculation and optimization methods.

### 4. Numerical Validations

During optimization process, the parameters (particles, maximum iteration) in the PSO and the MGC-MOPSO algorithms are set as (30, 200) and (50, 300), respectively. For reliability analysis, the confidence level is 0.95 and number of test designs is one million.

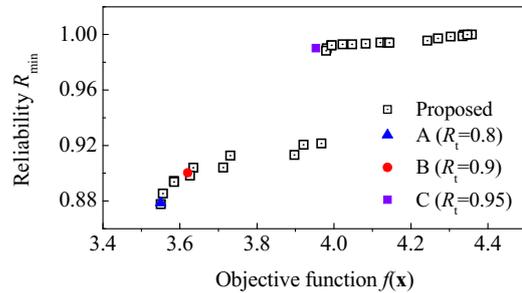
#### 4.1 Analytic example

To investigate performances of different RBOD algorithms, an analytic problem is selected as:

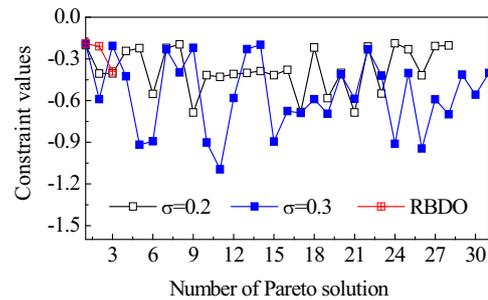
$$\begin{aligned}
 &\text{Minimize } f(\mathbf{x}) = \sin(3x_1^2) + \sin(3x_2^2) + x_1 + x_2 \\
 &\text{subject to } g_1(\mathbf{x}) = 1 - 1/(x_1^2 x_2) \leq 0 \\
 &\quad g_2(\mathbf{x}) = 1 - s^2 / 30 - t^2 / 120 \leq 0 \\
 &\quad g_3(\mathbf{x}) = 1 - 80/(x_1^2 + 8x_2 + 5) \leq 0
 \end{aligned} \tag{7}$$

where design space is  $0 \leq x_1, x_2 \leq 10$  ( $s = x_1 + x_2 - 5$  and  $t = x_1 - x_2 - 12$ ). The deterministic optimal solution is  $\mathbf{x} = (3.151308, 2.39507)^T$  [13]. For reliability analysis, uncertainty is  $\sigma=0.2$ .

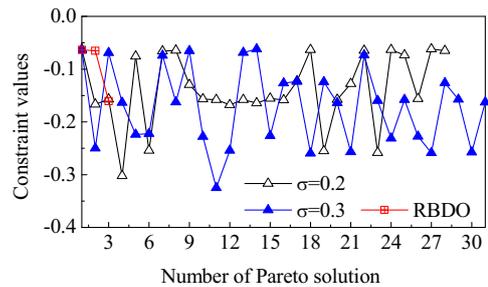
For each case of  $R_t=0.80, 0.90,$  and  $0.95$ , the optimization problem carried out 20 independent runs, among them, the best one is selected as the optimal



(a) Pareto optimal solutions  $\sigma=0.2$



(b) Comparison of constraint  $g_1(\mathbf{x})$



(c) Comparison of constraint  $g_2(\mathbf{x})$

**Fig. 4.** Optimization results of proposed RBOD algorithm

solution. Optimization results of the conventional RBOD are shown in Table 2. It can be seen that the RBOD finds almost similar solution with the deterministic/classical optimization when the target reliability is small. As the target reliability increases, the reliable optimal solution will be moved further away from the constraint boundaries. It is proved by values of the constraint functions, for example, the constraint value of  $g_1(\mathbf{x})$  at  $R_t=0.95$  as  $-0.38978$ , is much smaller than the one obtained at  $R_t=0.80$  as  $-0.18924$ . As in Fig. 4 (b) and (c), most of the constraint values at  $\sigma=0.3$  are smaller than those at  $\sigma=0.2$ . From last three columns, it is also validated that the reliable optimal designs obtained from the RBOD also satisfy the required reliability.

**Table 2.** Optimization results of conventional RBOD algorithm

$R_t$	Optimal design			Optimal performance				Reliability		
		$x_1$	$x_2$	$f(\mathbf{x})$	$g_1(\mathbf{x})$	$g_2(\mathbf{x})$	$g_3(\mathbf{x})$	$R(g_1(\mathbf{x}) \leq 0)$	$R(g_2(\mathbf{x}) \leq 0)$	$R(g_3(\mathbf{x}) \leq 0)$
0.80	A	3.151308	2.39507	3.55020	-0.18924	-0.06347	-1.34664	0.87825	0.90676	1.00000
0.90	B	3.166105	2.41243	3.62059	-0.20914	-0.06516	-1.33075	0.90042	0.91162	1.00000
0.95	C	3.151308	2.79895	3.95343	-0.38978	-0.16066	-1.14349	0.99026	1.00000	1.00000

Fig. 4 shows the optimization results of the proposed RBOD algorithm for multiple reliable solutions together with the RBOD results in Table 2. Since the objective function is multimodal, there are several gaps on the Pareto front as shown in Fig. 4. Every gap represents a change towards an inferior local optimum to satisfy the reliability requirement.

In addition, optimal solutions of the conventional RBOD under specified reliabilities such as design A, B, and C are close to some candidates on the Pareto front. In this sense, the conventional RBOD may be considered as a special case of the proposed multi-objective RBOD algorithm. Just from the number of solutions, by only one independent run, efficiency of the multi-objective RBOD is equivalent to several times of the conventional RBOD by taking the reliability condition as constraint. In other words, the proposed algorithm is more robust and useful than the conventional RBOD.

**4.2 Electromagnetic problem I - superconducting magnetic energy storage system**

In the 3-parameter superconducting magnetic energy storage system [2], combining energy requirement ( $E_0=180$  MJ) and minimal magnetic stray field ( $B_s$ ), objective function to be minimized and performance constraints guaranteeing superconductivity are formulated as:

$$f(\mathbf{x}) = \frac{B_s^2}{B_n^2} + \frac{|E(\mathbf{x}) - E_{ref}|}{E_{ref}}, \quad B_s^2 = \frac{1}{22} \sum_{i=1}^{22} B^2(i) \quad (8a)$$

$$g_i(\mathbf{x}) = |J_i| + 6.4 \cdot |B_{m,i}| - 54.0 \leq 0, \quad i = 1, 2 \quad (8b)$$

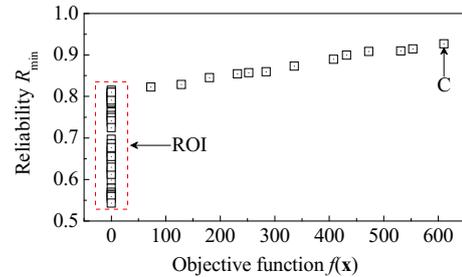
where  $B_n=3$  mT,  $B(i)$  is the magnetic flux density of the  $i$ th test point, and  $B_{m,i}$  is the maximum magnetic flux density of the  $i$ th coil. Fixed values of inner coil are  $[R_1, H_1, D_2]^T = [1.32, 2.14, 0.59]^T$  m [2].

There may be manufacturing tolerance in the geometric variables. In addition, the sources supplied by a current controller will keep in a certain range when compensating a perturbation so that they may deviate from the nominal values. Therefore, geometric variables  $\mathbf{x}=[R_2, H_2/2, D_2]^T$  are treated as uncertain ones while current densities  $\mathbf{J}$  are considered as uncertain parameters as listed in Table 3.

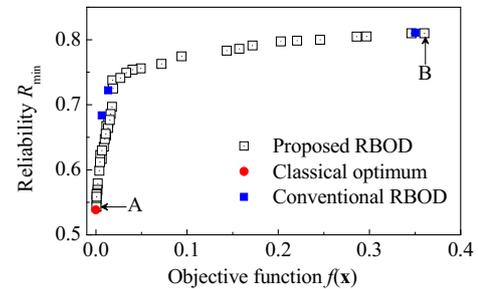
Table 4 compares optimization results of the RBOD under different target reliabilities with the deterministic/classical optimum. It is found that the classical optimum has very lower reliability, and it has higher possibility, in

**Table 3.** Uncertainties in design variables & parameters

Variable	$R_2$ [m]	$H_2/2$ [m]	$D_2$ [m]	$J_1$ [MA/m <sup>2</sup> ]	$J_2$ [MA/m <sup>2</sup> ]
$N(\mu, \sigma)$	$N(0, 0.01)$	$N(0, 0.01)$	$N(0, 0.01)$	$N(16.78, 0.23)$	$N(-15.51, 0.23)$



(a) Overall Pareto-front



(b) Pareto-front in the ROI

**Fig. 5.** Optimal results of the proposed RBOD algorithm for TEAM 22

this case 46.18%, to violate constraint  $g_1(\mathbf{x}) \leq 0$ . As the target reliability increases, the optimal reliable design gives a little worse objective value such as  $R^t = 0.7$  and  $0.8$ ; however, it locates further inside the feasible region with bigger margins to both constraints.

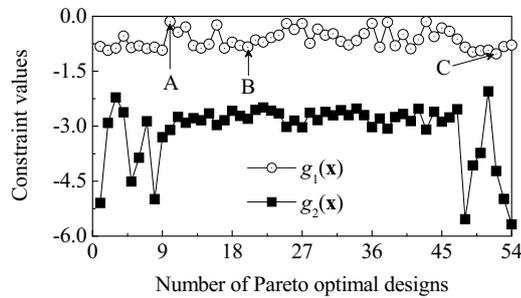
Fig. 5 shows the optimization result of the proposed reliability-based method. It is found that the Pareto-front includes the optimums obtained by the RBOD. In the region of interest (ROI), it is clear that design A (one of the extreme solutions) is very similar to the classical optimal design. The Pareto front also provides important information to make a balance between the objective function and minimum reliability according to different requirements. If the constraints are extremely critical, design C with bigger reliability may be selected although it has very poor performance. Design B in the ROI may be considered as a better solution in the general-purpose optimization since it makes a good trade-off between performance and reliability. Fig. 6 shows constraint values of each Pareto optimum,

**Table 4.** Optimal results of classical optimization and reliability-based design optimization <sup>a</sup>

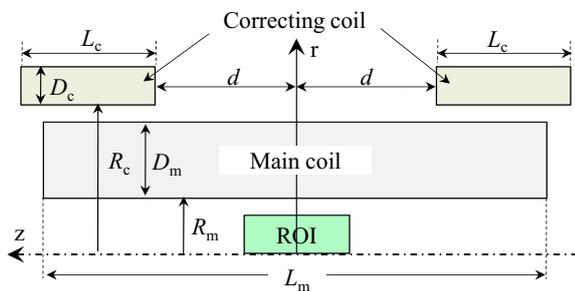
$R^t$	$R_2$	$H_z/2$	$D_2$	$f(\mathbf{x})$	$B_z^2 [T^2]$	$g_1(\mathbf{x})^b$	$R(g_1(\mathbf{x}) \leq 0)$
Classical	1.8127	1.4963	0.2458	$6.226 \times 10^{-5}$	$7.522 \times 10^{-11}$	-0.1191	0.5382
0.60	1.8176	1.4214	0.2567	$6.535 \times 10^{-3}$	$5.304 \times 10^{-9}$	-0.4762	0.6838
0.70	1.8121	1.4885	0.2519	$1.358 \times 10^{-2}$	$6.511 \times 10^{-8}$	-0.5918	0.7222
0.80	1.8064	1.7317	0.2392	$3.602 \times 10^{-1}$	$2.976 \times 10^{-6}$	-0.8841	0.8099

<sup>a</sup> The optimal design is selected among 20 independent runs.

<sup>b</sup> All designs have enough margins for constraint  $g_2(\mathbf{x}) \leq 0$ .



**Fig. 6.** Constraint values of each Pareto-optimal design



**Fig. 7.** Structure of Loney's correcting system

which reveals that a design with smaller margins for constraints will result in a lower reliability in Fig. 5 such as design A. It also illustrates that constraint  $g_1(\mathbf{x})$  is more sensitive to uncertainty than constraint  $g_2(\mathbf{x})$  for three-parameter problem.

### 4.3 Electromagnetic problem II – Loney's correcting System

As shown in Fig. 7, the Loney's correcting system includes a main coil and two correcting coils. The corresponding design problem is to determine the position and size of correcting coils so that it can generate a uniform magnetic flux density in the region of interest along the axis of a main solenoid [14, 15]. The homogeneity of magnetic field in the ROI is described as UB,

$$UB = \frac{B_{\max} - B_{\min}}{B_{\text{avg}}} \cdot 10^6 \text{ ppm} \quad (9)$$

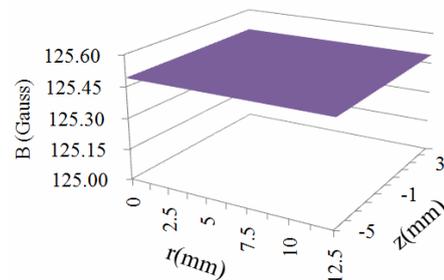
where  $B_{\min}$ ,  $B_{\max}$ , and  $B_{\text{avg}}$  are minimum, maximum, and average values of magnetic flux density in the ROI with size of 5mm×25mm. The design variable vector is  $\mathbf{x}=[R_c,$

**Table 5.** Design variables and uncertainty

	$R_c$ [mm]	$L_c$ [mm]	$d$ [mm]	$J_c$ [A/mm <sup>2</sup> ]
Min.	65	20	200	0
Max.	75	400	700	5
Uncertainty $\sigma$	0.5	0.5	5	0.05

**Table 6.** Result comparison

	Main coil	RBOD	MO-RBOD
$R_c$ [mm]	-	74.602411	74.646662
$L_c$ [mm]	-	87.428079	83.704947
$d$ [mm]	-	698.502256	689.361146
$J_c$ [A/mm <sup>2</sup> ]	-	4.991035	4.876507
$B_{\text{avg}}$ [Gauss]	125.2620	125.64069	125.4920
UB [ppm]	6.5433	1.8170	1.7969
Reliability	-	1.000	1.000



**Fig. 8.** Magnetic field distribution in ROI of MO-RBOD

$L_c, d, J_c]^T$  and values are set in Table 4, where  $J_c$  is the current density assigned to correcting coil. The fixed values are  $[R_m, L_m, D_m, D_c, J_m] = [55\text{mm}, 1400\text{mm}, 2\text{mm}, 2\text{mm}, 5\text{A/mm}^2]$ .

For a specified application, besides improving the magnetic field uniformity, the mean value of magnetic field also needs to be guaranteed. Therefore, in this paper, the multi-objective reliability-based optimal design problem is constructed as follows:

$$\begin{aligned} &\text{Maximize } f_1(\mathbf{x}) = B_{\text{avg}} \\ &\text{Maximize } f_2(\mathbf{x}) = R(UB \leq UB_0) \end{aligned} \quad (10)$$

where  $UB_0$  is the non-uniformity of main coil.

Similar as the foregoing examples, the MO-RBOD can supply a set of candidates, from the Pareto-front, one optimal design with  $R=1.00$  is selected to compare with other designs as shown in Table 5. The optimal of RBOD is obtained under the target reliability of 0.95. It can be seen that even the  $B_{\text{avg}}$  improvements of RBOD and MO-RBOD

are not too much compared with the main coil, their uniformities are improved obviously. Meanwhile, the reliability is guaranteed against uncertainty, which can be seen from Fig. 8.

## 5. Conclusion

A new reliability-based optimal design algorithm supplying a set of reliable solutions is suggested. The algorithm treats the uncertainty related constraints as additional independent objective functions to be maximized. It is validated that the Pareto-optimal designs from the proposed algorithm gives much more assistance to a designer in establishing the trade-off between the performance and reliability. The proposed algorithm can be combined with any reliability calculation methods such as reliability index approach and sampling-based methods.

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