

Covariance Matrix Synthesis Using Maximum Ratio Combining in Coherent MIMO Radar with Frequency Diversity

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Abstract – Reliable detection and parameter estimation of a radar cross section(RCS) fluctuating target have been known as a difficult task. To reduce the effect of RCS fluctuation, various diversity techniques have been considered. This paper presents a new method for synthesizing a covariance matrix applicable to a coherent multi-input multi-output(MIMO) radar with frequency diversity. It is achieved by efficiently combining covariance matrices corresponding to different carrier frequencies such that the signal-to-noise ratio(SNR) in the combined covariance matrix is maximized. The value of a synthesized covariance matrix is assessed by examining the phase curves of its entries and the improvement on direction of arrival(DOA) estimation.

Keywords: Covariance matrix, RCS, Frequency diversity, DOA, Multicarrier, MIMO

1. Introduction

In order to ensure target detection and its parameter estimation, enough output SNR is required. However, the required SNR can not be guaranteed for a RCS fluctuating target even with sufficient transmitting power [1]. To reduce the RCS fluctuating target effect, a widely separated MIMO radar was proposed [2]. It exploits spatial diversity such that target signals impinging on antenna elements are statistically independent and the combined output reduces the RCS fluctuation [3, 4]. [5, 6] showed that detection and estimation performance is further improved using additional use of different carrier frequencies on widely separated MIMO structure, conclusively exploiting spatial-frequency diversity. However, due to the wide separation between receiving elements, a widely separated MIMO radar can not but face synchronization problem and thus can not achieve coherent processing [7].

Aside from widely separated MIMO radar, coherent MIMO radar with collocated antenna elements was proposed. It uses waveform diversity to attain higher angular resolution by synthesizing virtual array aperture and is known as offering better detection performance than widely separated MIMO radar at low detection probabilities [4]. However, coherent MIMO radar is basically not useful for mitigating the effect of target RCS fluctuation because of the spatially collocated deployment of antenna elements [8, 9]. In many works concerning coherent MIMO radar, the application of

frequency diversity have been considered, expecting it to weaken the RCS fluctuation. [7] proved that via frequency diversity, estimation accuracy of target's parameter can be improved, presenting the Cramer-Rao bound. And [10] gave some analysis about range resolution and compound angle-range beampattern in the case where frequency diversity is incorporated. Recently, [11] addressed the waveform design using multi-carrier with the intention that the designed waveform should have low-cross correlation and low-autocorrelation sidelobes. [12] optimized multicarrier waveform based on the mutual information criterion. Also in monostatic environment, covariance estimation using Toeplitz structure for clutter suppression are proposed [13].

The above-mentioned researches relating to coherent MIMO radar, the improvement on detection and estimation performance was attained by beamforming, waveform design, etc.

It has been well-known that to estimate target's parameters such as DOA, a covariance matrix containing target signal component of high SNR is definitely needed. Thus, this paper first focuses on synthesizing a covariance matrix in coherent MIMO radar with frequency diversity in the way that it maximizes SNR and later, shows how it affects DOA estimation performance.

In section 2, we introduce a system model that describes the mathematical form for covariance matrices corresponding to different carrier frequencies. In section 3, we present a method to synthesize a covariance matrix out of the covariance matrices. In order to assess the effectiveness of a synthesized covariance matrix, simulation results are given in section 4.

2. System Model

Consider a radar system with a receiver of an M-element

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array. A radar system could operate in either monostatic or bistatic mode. A transmitter sends a multi-carrier waveform consisting of N carrier frequencies. The carrier frequency interval in a multi-carrier waveform is constant and designated large enough for the target echoes corresponding to different frequency components to be independent from each other. To assure independence between the frequency components, the frequency interval should be at least $c/(2L(\theta))$, where $L(\theta)$ is the length of the target projected along the radar boresight [1]. The transmitted waveform $x(t)$ can be written as

$$x(t) = \sum_{i=1}^N x_i(t) = \text{Re} \left[\sum_{i=1}^N \tilde{x}_i(t) e^{j2\pi f_i t} \right] \quad (1)$$

where f_c is the carrier frequency, and $\tilde{x}_i(t)$ is the complex amplitude of the frequency component at $f_c + f_i$. Thus, $\tilde{x}_i(t)$ can be expressed as

$$\tilde{x}_i(t) = g(t) e^{j2\pi f_i t} \quad i = 1, 2, \dots, N \quad (2)$$

where $g(t)$ is the pulse amplitude. Let p_{f_i} denote the power of $\tilde{x}_i(t)$, that is, $E[x_i(t)x_i^H(t)] = p_{f_i}$. The duration of $x(t)$ is assumed to be long enough so that each $x_i(t)$ is regarded as a narrowband signal. We further assume that at each element of a receiver array, the N frequency components of a received echo can be separated by bandpass filtering, filtered signal components be subsequently converted into baseband signals and a receiver regroups the baseband-converted echoes according to their frequencies. Let $\mathbf{r}_i(t)$ denote the regrouped signal vector of the $f_c + f_i$ frequency component. Then, $\mathbf{r}_i(t)$, an complex $M \times 1$ vector whose entries come from their respective array elements, can be written as

$$\mathbf{r}_i(t) = \mathbf{a}_i^H c_i \tilde{x}_i(t - \tau) + \mathbf{n}_i \quad i = 1, 2, \dots, N \quad (3)$$

where $\mathbf{a}_i \in \mathbb{C}^M$, $c_i \in \mathbb{C}$ and $\mathbf{n}_i \in \mathbb{C}^M$ denote a steering vector, target complex amplitude, and AWGN noise vector where $\mathbf{n}_i \sim CN(0, \sigma_n^2)$, respectively, and τ is the travel time between a transmitter and a first/reference element in the receiver array. For the target echo impinging on the receiver array from aspect angle θ , \mathbf{a}_i becomes

$$\mathbf{a}_i^H = \left[1 \quad e^{j\frac{2\pi}{\lambda_i} d \sin \theta} \quad e^{j\frac{2\pi}{\lambda_i} 2d \sin \theta} \quad \dots \quad e^{j\frac{2\pi}{\lambda_i} (M-1)d \sin \theta} \right] \quad (4)$$

where d is the spatial distance between elements of the receiver array. To avoid a grating lobe, d should be less than $\frac{\lambda_{\min}}{2}$, where λ_{\min} is a wavelength corresponding to $f_c + f_N$, which is the highest frequency out of N carrier frequencies. The received signal power for $\mathbf{r}_i(t)$ can be given by

$$|\mathbf{r}_i(t)|^2 = \frac{1}{N} \text{Tr}(\mathbf{r}_i(t)\mathbf{r}_i^H(t))$$

$$= \frac{1}{N} \text{Tr}(\mathbf{a}_i^H E[c_i c_i^H] \mathbf{a}_i p_{f_i} + E[\mathbf{n}_i \mathbf{n}_i^H]) \quad (5)$$

$$= g_i p_{f_i} + \sigma_n^2$$

where Tr denotes a trace operator, g_i specifies the average RCS defined as $E[c_i c_i^*] = g_i$ and σ_n^2 is AWGN noise variance. A covariance matrix constructed with $\mathbf{r}_i(t)$, $\mathbf{R}_{f_i} \in \mathbb{C}^{M \times M}$ can be written as

$$\mathbf{R}_{f_i} = E[\mathbf{r}_i(t)\mathbf{r}_i^H(t)] = g_i p_{f_i} \mathbf{a}_i^H \mathbf{a}_i + \sigma_n^2 \mathbf{I} \quad (6)$$

$$i = 1, 2, \dots, N$$

where \mathbf{I} is the $M \times M$ identity matrix. Since one covariance matrix can be constructed for its respective regrouped signal vector, N covariance matrices $\{\mathbf{R}_{f_i}, i = 1, 2, \dots, N\}$ can be constructed. If there were no AWGN, $\mathbf{R}_{f_i}(j, k)$, an entry in row j and column k of \mathbf{R}_{f_i} , can be written as

$$\mathbf{R}_{f_i}(j, k) = \mathbf{B}_{(j,k)} e^{j2\pi f_i K_\theta(j,k)} \quad j, k = 1, 2, \dots, M \quad (7)$$

where $\mathbf{B}_{(j,k)} \in \mathbb{R}$ and $2\pi f_i K_\theta(i, j)$, respectively, are the amplitude and the phase of $\mathbf{R}_{f_i}(j, k)$ and $K_\theta(j, k) = (k + j - 2)d \sin \theta$.

3. Synthesis of a Covariance Matrix

3.1 Phase Synchronization

To implement some conventional narrowband DOA algorithms such as Capon, MUSIC, etc., a covariance matrix constructed with a narrowband echo is needed. Thus, the goal of the proposed algorithm is to synthesize a covariance matrix with N covariance matrices mentioned in the previous section.

In the proposed algorithm, a covariance matrix is synthesized by two steps. The first step is to synchronize/equate the phases of the entries in the same row/column position in all the N covariance matrices. This is accomplished by scaling on the phases of the entries. Suppose there were no AWGN in target echoes. For $\mathbf{R}_{f_i}(j, k)$, for example, we multiply its phase $2\pi f_i K_\theta(i, j)$ by f_o / f_i so that the resultant phase becomes $2\pi f_o K_\theta(i, j)$. Here, f_o could be any f_i among N frequencies or any hypothetical frequency that plays a role of reference frequency. This scaling is applied to all the entries in \mathbf{R}_{f_i} . Likewise, we multiply f_o / f_i for all the entries in \mathbf{R}_{f_j} . Let $\hat{\mathbf{R}}_{f_i} \in \mathbb{C}^{M \times M}$ be a covariance matrix obtained after the phase scaling on \mathbf{R}_{f_i} . Then, the (j, k) entry in $\hat{\mathbf{R}}_{f_i}$ becomes

$$\hat{\mathbf{R}}_{f_i}(j, k) = \mathbf{B}_{(j,k)} e^{j2\pi f_o K_\theta(j,k)} \quad (8)$$

Now, the phases of entries in the same row and column

position in all covariance matrices, $\widehat{\mathbf{R}}_f \in \mathbb{C}^{M \times M \times N}$, $\widehat{\mathbf{R}}_f = [\widehat{\mathbf{R}}_{f_1}, \widehat{\mathbf{R}}_{f_2}, \dots, \widehat{\mathbf{R}}_{f_N}]$ would be equal if there were no AWGN in target echoes. It is noted that the scaled phase values are equivalent to the phase values in a covariance matrix that would be obtained from the echoes to be receivable if a hypothetical single-carrier waveform centered at $f_c + f_o$ were transmitted. To sum up, the first step is to synchronize the phase of the entries, which is needed for the matrix addition to perform in the next step.

The phase discrepancy definitely exists, and it would matter if the channel noise were heavier or the number of snapshots for calculating the covariance matrices were not sufficient. The second step is to effectively combine N covariance matrices to synthesize a desired covariance matrix, which is called $\mathbf{R}_o \in \mathbb{C}^{M \times M}$ in the following discussions. A very simple and intuitive way to synthesize \mathbf{R}_o is to add all covariance matrices in $\widehat{\mathbf{R}}_f$, that is, $\mathbf{R}_o = \sum_{i=1}^N \widehat{\mathbf{R}}_{f_i}$ known as maximum likelihood estimation, or equivalently, to average the entries in the same position. The amplitudes of the entries in the matrices actually have no DOA information. However, as will be clear in section 4, the synthesized \mathbf{R}_o obtained through adequately using 0074he amplitude information will give better performance. The section 3.2 will explain how to determine the weighting factors, $\mathbf{w} \in \mathbb{R}^M$, $\mathbf{w} = [w_1, w_2, \dots, w_N]$, by which phase-scaled covariance matrices are combined as

$$\mathbf{R}_o = \sum_{i=1}^N w_i \widehat{\mathbf{R}}_{f_i} \quad (9)$$

3.2 Determination of weighting factors

Before delving into the derivation of the weighting factors \mathbf{w} , we briefly review Capon's algorithm. Consider a simple narrowband scenario where a narrowband echo reflected from a single target at the aspect angle θ_1 is impinging on a receiver array. Assume a covariance matrix having target signature, $\mathbf{R}_x \in \mathbb{C}^{M \times M}$ is constructed by a sufficient number of snapshots. Then, all the diagonal entries of \mathbf{R}_x are approximately $p_s + \sigma^2$, where p_s and σ^2 are the target signal power and noise variance, respectively [1]. The amplitudes of all the off-diagonal entries in \mathbf{R}_x are approximately p_s . By the Capon algorithm, the spatial power spectrum $p(\theta)$, which is

$$p(\theta) = \frac{1}{\mathbf{a}_i^H(\theta) \mathbf{R}_x^{-1} \mathbf{a}_i(\theta)} \quad (10)$$

, is obtained by varying θ in $\mathbf{a}_i(\theta)$, which is equivalently defined as in (4). By mathematical manipulation, we have

$$p(\theta) = \frac{(2\sigma^2 p_s + \sigma^4)}{2(p_s + \sigma^2 - p_s \cos(\theta - \theta_1))} \quad (11)$$

Thus, the peak of $p(\theta)$ occurs at $\theta = \theta_1$, and its peak value is $p_s + \sigma^2 / 2$. This implies that in order to maximize the peak value for a fixed p_s, σ^2 or SNR (which is $\frac{p_s}{\sigma^2}$) should be maximized. Moreover, a higher peak value in turn suggests more excellent DOA estimation.

Let us revert to our problem to determine \mathbf{w} shown in (9). By simple manipulation to put (4) into (6), the diagonal entries in $\widehat{\mathbf{R}}_{f_i}$ are roughly all $g_i p_{f_i} + \sigma_n^2$, which is equivalent to the sum of the signal power of the $f_c + f_i$ frequency components and noise power. Moreover, the amplitudes of all the off-diagonal entries in $\widehat{\mathbf{R}}_{f_i}$ are roughly $g_i p_{f_i}$, which is the signal power. As the values of the diagonal entries and the amplitude of off-diagonal entries in $\widehat{\mathbf{R}}_f$ can be similarly given, the diagonal entries in \mathbf{R}_o are roughly all $\sum_{i=1}^N w_i (g_i p_{f_i} + \sigma_n^2)$, and the amplitudes of the off-diagonal entries are roughly all $\sum_{i=1}^N w_i g_i p_{f_i}$. It is noted that the forms of entry values in \mathbf{R}_o are analogous to those in the illustrated covariance matrix \mathbf{R}_x which was used to explain the Capon's algorithm. Hence, we may select \mathbf{w} such that the SNR of $\sum_{i=1}^N w_i (g_i p_{f_i} + \sigma_n^2)$ is maximized, which reduces to a diversity combining problem.

For an independent AWGN channel, as we are faced with, the maximum ratio combining (MRC) has been known the optimum combiner [13]. The theory of the MRC dictates that the optimum \mathbf{w} is the one proportional to the SNR of an associated signal component. Thus, we may choose \mathbf{w} in such a way that:

$$w_i = g_i P_{f_i} / \sigma_n^2 \quad i = 1, 2, \dots, N \quad (12)$$

In the following discussions, the averaging method in which all entries in \mathbf{w} have the same values will be called an equal gain combining (EGC) method as opposed to the MRC.

4. Simulation

A target is simulated as a collection of 30 point scatterers that are randomly positioned along a 20-m line. The reflectivity function of the point scatterers is modelled as having complex Gaussian distribution. In order to assure adequate frequency interval of a multi-carrier waveform, the correlation between two adjacent baseband-converted echoes of different frequencies is examined.

As shown in Fig. 1, the frequency interval of 50 MHz is good enough to assume statistical independence. Thus, a transmitter is assumed to send a constant amplitude 4-carrier waveform whose carrier frequencies are 10GHz, 10.05GHz, 10.1GHz, and 10.15GHz. A receiver with 8-elements makes snapshots for a target echo to calculate the

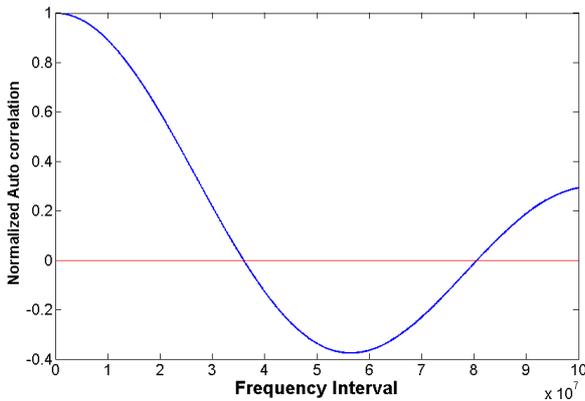


Fig. 1. Correlation versus frequency interval

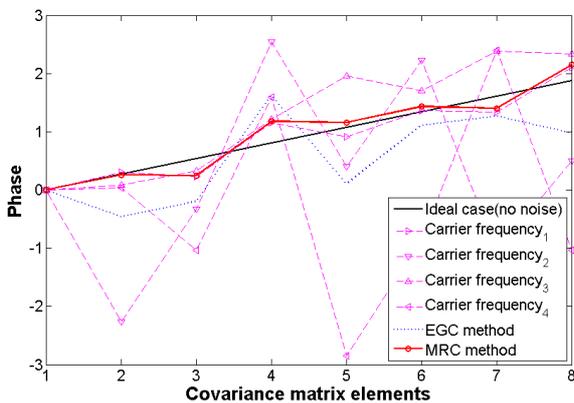


Fig. 2. Phase curves in first row entries

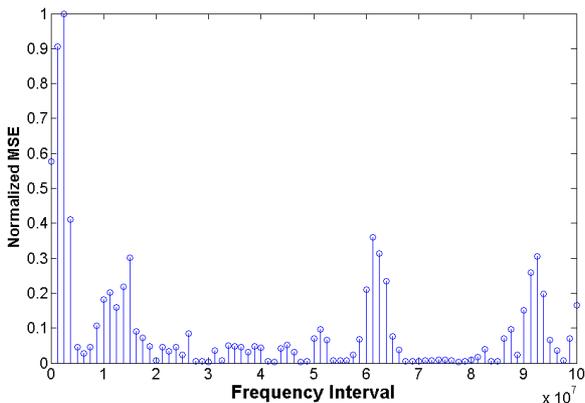


Fig. 3. MSE of DOA estimation versus frequency interval

covariance matrix $\mathbf{R}_{f_i} (i=1,2,3,4)$. $\hat{\mathbf{R}}_{f_i}$ is calculated from \mathbf{R}_{f_i} for $f_c + f_o = 10\text{GHz}$. In noise-free channel, the phase of $\hat{\mathbf{R}}_{f_i} (j, k)$ is $2\pi f_o K_\theta(j, k) = 2\pi f_o (k + j - 2)d \sin \theta$. Thus, the phases of the entries in the first row, which are $\hat{\mathbf{R}}_{f_i} (1, k)$, should increase as k increases if there were no AWGN.

Fig. 2 shows the phase curves of the first row entries in all $\{\hat{\mathbf{R}}_{f_i}\}$. Because of AWGN, all the curves fluctuate but increase gradually. We next calculate by the EGC and the MRC. Although both reduce the phase fluctuation, the

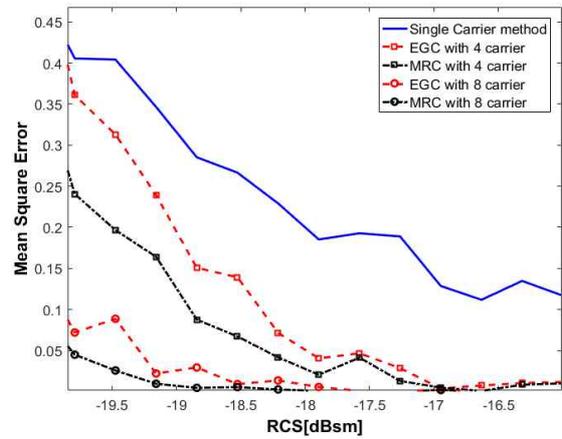


Fig. 4. MSE of DOA estimation in Swerling I model

latter exhibits a phase curve much closer to the ideal one obtainable in noise-free channel. Hence, we may expect the \mathbf{R}_o by the MRC to give better performance in DOA estimation than the \mathbf{R}_o by the EGC does.

Next, using the \mathbf{R}_o from the MRC, the DOA is calculated using the Capon algorithm. The frequency interval of the 4-carrier waveform is varied to examine its effect on the DOA estimation. Fig. 3 shows the mean square error (MSE) of the DOA as the function of the frequency interval. The performance in DOA estimation deteriorate further when the frequency interval causing more correlation is selected. This is intuitively anticipated in that the diversity-combining, whether it is the MRC or the EGC, produces higher SNR output as the signals from different frequency channels are more independent.

In the next simulation, a target RCS is assumed to vary according to the Swerling I model. Hence, the only parameter determining its probability density function is the mean value of target RCS. \mathbf{R}_o is calculated by the EGC, the MRC, and the so called single-carrier method. In the single-carrier method in which a single-carrier waveform is transmitted, the signal power of the single-carrier waveform is chosen to be such that the target signal powers from three methods are the same. Fig. 4 shows the MSE of the DOA estimation by three values of \mathbf{R}_o as the function of mean RCS. The MRC method outperforms the other methods in estimating the DOA.

5. Conclusion

We have presented a new method to synthesize a covariance matrix that combines the covariance matrices associated with transmitting carrier frequencies, which results in mitigating the target RCS fluctuation. To implement the proposed method, the phase synchronization of entries in the same row and column for all the covariance matrices should be done first. In the second step, they are combined to maximize the SNR of the synthesized

covariance matrix. We gave some analysis supporting that the MRC can achieve that purpose and verified it through simulations. To compare it with the EGC and the conventional single-carrier method, we examined the phase curves of the matrix entries and the DOA estimation performances. Simulation results showed that the MRC outperformed the other methods, exhibiting the lowest MSE of the DOA estimation. Moreover, it was shown that the DOA estimation results highly depends on the frequency interval of the multi-carrier waveform, which affects the independence between different frequency components of target echoes.

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