

# Adaptive Sliding Mode Traffic Flow Control using a Deadzoned Parameter Adaptation Law for Ramp Metering and Speed Regulation

Xin Jin\*, Myunghwan Eom\*\* and Dongkyoung Chwa†

**Abstract** – In this paper, a novel traffic flow control method based-on ramp metering and speed regulation using an adaptive sliding mode control (ASMC) method along with a deadzoned parameter adaptation law is proposed at a stochastic macroscopic level traffic environment, where the influence of the density and speed disturbances is accounted for in the traffic dynamic equations. The goal of this paper is to design a local traffic flow controller using both ramp metering and speed regulation based on ASMC, in order to achieve the desired density and speed for the maintenance of the maximum mainline throughput against disturbances in practice. The proposed method is advantageous in that it can improve the traffic flow performance compared to the traditional methods using only ramp metering, even in the presence of ramp storage limitation and disturbances. Moreover, *a priori* knowledge of disturbance magnitude is not required in the process of designing the controller unlike the conventional sliding mode controller. A stability analysis is presented to show that the traffic system under the proposed traffic flow control method is guaranteed to be uniformly bounded and its ultimate bound can be adjusted to be sufficiently small in terms of deadzone. The validity of the proposed method is demonstrated under different traffic situations (i.e., different initial traffic status), in the sense that the proposed control method is capable of stabilizing traffic flow better than the previously well-known Asservissement Lineaire d'Entree Autoroutiere (ALINEA) strategy and also feedback linearization control (FLC) method.

**Keywords:** Traffic flow control, Ramp metering, Speed regulation, Adaptive sliding mode control, Deadzoned parameter adaptation, Disturbance

## 1. Introduction

Growing traffic demand has led to a steady increase of recurrent and non-recurrent freeway congestion. Freeway congestion not only reduces the nominal capacity of the freeway infrastructure but also causes excessive delays, increases fuel consumption and environmental pollution, and deteriorates traffic safety. Therefore, traffic control should be adopted to relieve the congestions and improve the efficiency of freeways. Various traffic control methods have been employed to address the issue of traffic congestion, such as the ramp metering (RM), speed control, and route guidance (RG). While RM regulates the number of vehicles entering the freeway at freeway entrance ramps in such a way that the traffic flow is maintained around the desired density level, speed control aims to ensure harmony in the interactions between vehicles and the road environment [1], and route guidance relieves traffic jams by identifying the best route to be followed towards a desired destination.

Among these freeway traffic control approaches, two

major ones include the RM and speed control. First, RM can be categorized into the reactive strategy and proactive strategy approaches. The reactive strategy aims to maintain the traffic density close to the pre-specified set values using real-time measurements, and the proactive strategy aims to formulate optimal traffic conditions for an entire freeway network. Before performing the optimization, traffic state and demand prediction is required [2]. From the view point of execution, there are two types of RM, namely, local metering and coordinated metering. While the local metering approach considers the local upstream and downstream traffic information to control the local on-ramps, the coordinated metering approach considers system-wide vehicle interaction and traffic information of an entire network to control all on-ramps within the region [3]. The traffic responsive RM control strategy is a popular way to maximise the freeway traffic capacity including ALINEA, which is a well-known local RM control method [4], and its variants [2, 5]. A variety of optimal control methods have been employed to handle the coordinated ramp control problem to attain the optimal dynamic performance instead of a traditional static traffic flow performance [6, 7, 8]. On the other hand, variable speed limits (VSL) displayed on road-side variable message signs (VMS) is one of the most extensively studied speed control methods. It is mainly used to limit speed to enhance traffic

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safety through the homogenisation of speeds of individual vehicles [9, 10]. In addition, another speed control method called speed regulation exists that calculates the desired speed commands (which can be communicated to the vehicles on the roadway) to be followed by the vehicles in the freeway in order to achieve the desired traffic density [11, 12]. In the case of speed regulation, the driver's actions are replaced by speed control law when the drivers receive and follow the speed command messages sent from the control center through real-time communications [13]. It should be noted here that the speed regulation can be implemented owing to the basic principles of wireless vehicle-to-infrastructure communication [14, 15]. These principles state that vehicles can receive messages, including traffic information, warnings, alarms, and speed commands. An algorithm combining VSL and coordinated RM (CRM) with an optimal control approach is described in [16]. The model predictive control (MPC) approach is also used for traffic flow control with VSL [17] and with both VSL and RM [18]. The iterative learning control-based RM strategy for freeway traffic control is studied in [19]. A coordinated traffic responsive ramp control strategy based on feedback control and neural networks has been developed in [20]. Another approach based on fuzzy logic for RM has been proposed in [21], where a fuzzy logic control method shows the robustness and the satisfactory tracking performance.

Nevertheless, the aforementioned freeway control strategies are limited in handling a number of issues. Specifically, since traffic flow behavior exhibits nonlinear and real-time characteristics, the influence of the dynamical randomness on traffic flow should be taken into account in the design of the traffic controller. However, linear control methods such as ALINEA may not be able to control the nonlinear ramp system [22]. In particular, disturbances practically occur in both density and speed owing to large modeling uncertainties. Thus, they should be effectively compensated to avoid corresponding performance degradations. Therefore, it is necessary to develop nonlinear robust traffic flow control methods. Since the sliding mode control (SMC) method [23] can be used for many practical systems owing to the robustness against system uncertainties and external disturbances, it can be effectively employed in traffic flow control to solve the aforementioned problems. In recent years, more results on the application of the SMC method to traffic flow control have been studied. For example, several SMC-based RM methods have been developed in [24, 25, 26]. Unfortunately, most of them are based on the macroscopic one-dimensional traffic flow model and are focused only on RM control and not speed regulation. In particular, ramp storage limitation is not included in simulations and the upper bounds of the disturbances are usually assumed to be known, which may not be the case in practice.

In this paper, we propose a novel traffic flow control approach, based on RM and speed regulation, using an

adaptive sliding mode control (ASMC) along with a deadzoned parameter adaptation law to achieve the desired density and speed. First of all, unlike the ALINEA method which cannot compensate for the disturbances, the proposed method can maintain the expected performance. In addition, unlike other SMC-based methods, a prior knowledge of the upper bounds of the disturbances is not necessary and simply needs to be estimated using the proposed deadzone-based adaptation law. Additionally, an adaptive sliding mode traffic control law is constructed in such a way that sliding surfaces of both density and speed can be reached in finite time, even in the presence of disturbances in density and speed dynamics. The application of the ASMC method for both ramp metering and speed regulation of the traffic flow model has not been studied to the best of authors' knowledge and thus the advantage of the ASMC method with a deadzoned parameter adaptation law is noticeable in that the disturbance present in traffic flow model can be effectively compensated without involving the chattering phenomenon and thus the traffic flow control performance can be maintained even in the presence of both disturbances and input constraint. The stability analysis of the overall traffic flow control system is performed using the Lyapunov stability method [27], and simulation results demonstrate the validity of the proposed method in the presence of disturbances and metered on-ramp outflow constraint.

The rest of this paper is organized as follows. In Section II, a stochastic macroscopic traffic flow model is presented and the traffic flow control problem is formulated. An ASMC method for both RM and speed regulation, and the stability analysis of the overall traffic flow control system are presented in Section III. In Section IV, simulation results are provided to demonstrate the proposed method. Finally, the conclusion is presented in Section V.

## 2. Stochastic Macroscopic Traffic Flow Model

In this section, a second order macroscopic traffic flow model [19, 28, 29] is selected to describe the traffic freeway system. Its effectiveness has been demonstrated to provide a good trade-off between simulation speed and accuracy.

A freeway link is first divided into  $N$  sections (indicated

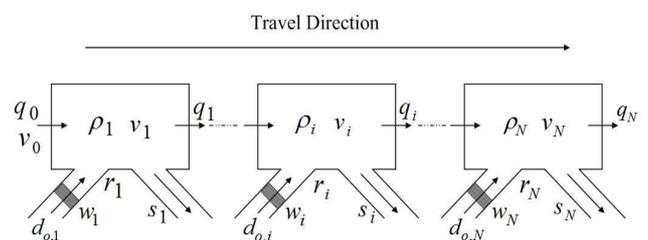


Fig. 1. A freeway system with on/off ramps and queue length

by the subscript  $i$ ), of length  $L$  (typically 500m), where an on-ramp and/or off-ramp may exist. A traffic model of each section is characterized by three traffic variables: traffic density  $\rho_i(t)$ , space mean speed  $v_i(t)$ , and traffic flow  $q_i(t)$  as shown in Fig. 1.

Considering the influence of the density and the speed disturbances, the equations of the stochastic nonlinear model for each section are given as follows:

$$\frac{d}{dt} \rho_i(t) = \frac{1}{L_i} [q_{i-1}(t) - q_i(t) + r_i(t) - s_i(t)] + \xi_{\rho,i}(t) \quad (1)$$

$$\begin{aligned} \frac{d}{dt} v_i(t) = & \frac{1}{\tau} [V[\rho_i(t)] - v_i(t)] + \frac{1}{L_i} v_i(t) [v_{i-1}(t) - v_i(t)] \\ & - \frac{v}{\tau L_i} \frac{[\rho_{i+1}(t) - \rho_i(t)]}{[\rho_i(t) + \kappa]} + \xi_{v,i}(t) \end{aligned} \quad (2)$$

$$q_i(t) = \rho_i(t) v_i(t) \quad (3)$$

$$V[\rho_i(t)] = v_f \left\{ 1 - \left[ \frac{\rho_i(t)}{\rho_{\max}} \right]^l \right\}^m \quad (4)$$

Here, Eqs. (1), (2), and (3) are the conservation, dynamic, and flow equations, respectively. In the following variables, the subscript and the argument denote the variables of section  $i$  and time  $t$ , respectively.  $\tau$ ,  $v$ ,  $\kappa$ ,  $v_f$ ,  $\rho_{\max}$ ,  $l$ , and  $m$  are model parameters which are used to represent the freeway characteristics. The third term in the right hand side of Eq. (2) (i.e.,  $\frac{v}{\tau L_i} \frac{[\rho_{i+1}(t) - \rho_i(t)]}{[\rho_i(t) + \kappa]}$ ) is the anticipation term that reflects the effect of downstream traffic density on the mean speed evolution. While this term is no longer adjustable by human control, it can be adjusted by the control law determined in the speed regulation input  $u_{v,i}(t)$ , as in [11, 12].

To describe the density and the speed entering the first section and leaving the last section, the following equations of the boundary condition can be introduced.

$$\rho_0(t) = q_0(t) / v_1(t) \quad (5)$$

$$v_0(t) = v_1(t) \quad (6)$$

$$\rho_N(t) = \rho_{N-1}(t) \quad (7)$$

$$v_N(t) = v_{N-1}(t). \quad (8)$$

In addition, for origin links (i.e., an on-ramp) which receive a traffic demand  $d_{o,i}(t)$  and forward it into the mainstream, a simple queue model is used as follows:

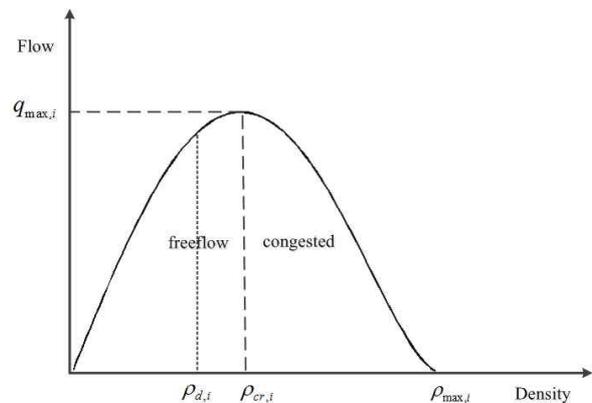
$$\frac{d}{dt} w_i(t) = d_{o,i}(t) - q_{o,i}(t). \quad (9a)$$

Here,  $w_i(t)$  is the queue length at the origin  $o$ ,  $d_{o,i}(t)$  is the demand flow at  $o$ , and  $q_{o,i}(t)$  is the outflow at the origin link  $o_i$ . For a metered on-ramp,  $q_{o,i}(t)$  corresponds to the control input variable  $u_i(t)$  and is subject to the input constraint  $u_{\min,i} < u_i(t) < u_{\max,i}(t)$ , where  $u_{\min,i}$  and  $u_{\max,i}(t)$  are the minimum and maximum values entering the mainstream from the on-ramp, respectively. Since the outflow  $q_{o,i}(t)$  depends on the traffic flow conditions of the mainstream section, the metered on-ramp outflow constraint is determined as follows:

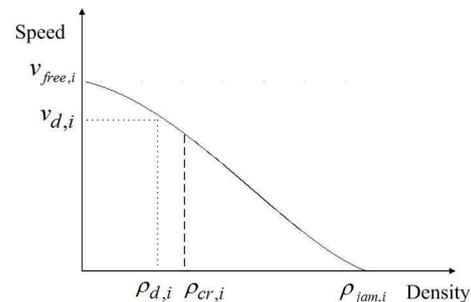
$$\begin{aligned} u_{\max,i}(t) = & \min \left\{ d_{o,i}(t) + w_i(t) / T, Q_{sat} \cdot \min \left\{ 1, \frac{\rho_{\max,i} - \rho_i(t)}{\rho_{\max,i} - \rho_{cr,i}} \right\} \right\} \\ u_{\min,i} = & 0 \end{aligned} \quad (9b)$$

where  $T$  is the time step,  $Q_{sat,i}$  (veh/h) is the on-ramp's flow capacity, and  $\rho_{\max,i}$  (veh/km/lane) is the maximum density of the mainstream section.

Since both RM and speed regulation should be employed in the proposed method, the density  $\rho_i(t)$  and the speed  $v_i(t)$  of the freeway section are selected as the state variables in order to evaluate the mainline throughput, according to Eq. (3). Considering that the desired density,  $\rho_{d,i}(t)$  can be obtained from the fundamental diagram as in Fig. 2(a), the desired density  $\rho_{d,i}(t)$  should be selected



(a) Flow-density curve



(b) Speed-density curve

Fig. 2. Fundamental diagram of traffic flow

to be lightly smaller in value than the critical density  $\rho_{cr,i}(t)$  to avoid congestion while keeping the maximum throughput of the mainstream. The desired speed  $v_{d,i}$  can be obtained from Fig. 2(b). Since the traffic flow control problem can be formulated into an output tracking and a disturbance rejection problem, the control objective is to seek an appropriate on-ramp metering  $r_i(t)$  and a speed regulation  $u_{v,i}(t)$  so that the actual traffic density and speed in the controlled section can converge to the desired traffic density  $\rho_{d,i}(t)$  and to a speed  $v_{d,i}(t)$ , even in the presence of disturbances. It should be noted that  $V[\rho_d] = v_d$  represents the static relationship between the desired speed and the desired density in a fundamental diagram that is used to obtain just the desired speed. In this paper, the speed dynamic equation [Eq. (2)] is employed, unlike many previous studies where the static relationship  $v_i(t) = V[\rho_i(t)]$  was used. In addition, speed regulation is considered, since it can increase the density when on-ramp metering cannot supply enough vehicles entering the mainstream, and since it can also improve the density and speed tracking performance, as seen from the simulation results in Section IV.

### 3. Adaptive Sliding Mode Traffic Flow Control

Owing to the simple design procedure and the robustness against system uncertainties and external disturbances, the SMC method has been widely employed in the control area. In this section, we propose an ASMC method to handle traffic flow control problems. For section  $i$ , we first define the desired density  $\rho_{d,i}$  and the desired speed  $v_{d,i}$  according to the fundamental diagram based on the equation  $V[\rho_{d,i}] = v_{d,i}$ . Let  $e_{1,i}$  and  $e_{2,i}$  denote the density and speed tracking errors, respectively, given by

$$e_{1,i} = \rho_{d,i} - \rho_i, \quad e_{2,i} = v_{d,i} - v_i. \tag{10}$$

Then, the error dynamic equations can be obtained from Eqs. (1)-(4) as follows:

$$\begin{aligned} \dot{e}_{1,i}(t) &= -\frac{1}{L_i} [q_{i-1} - (\rho_{d,i} - e_{1,i})(v_{d,i} - e_{2,i}) + u_{1,i}] - \xi_{\rho,i}(t) \\ &= -\frac{1}{L_i} q_{i-1} + \frac{1}{L_i} (\rho_{d,i} v_{d,i} - v_{d,i} e_{1,i} - \rho_{d,i} e_{2,i} + e_{1,i} e_{2,i}) \\ &\quad - \frac{1}{L_i} u_{1,i} - \xi_{\rho,i}(t) \\ &= \frac{1}{L_i} e_{1,i} e_{2,i} - \frac{1}{L_i} v_{d,i} e_{1,i} - \frac{1}{L_i} \rho_{d,i} e_{2,i} + \frac{1}{L_i} \rho_{d,i} v_{d,i} \\ &\quad - \frac{1}{L_i} q_{i-1} - \frac{1}{L_i} u_{1,i} - \xi_{\rho,i}(t) \\ &= f_{1,i}(e_{1,i}, e_{2,i}, \rho_{d,i}, v_{d,i}, q_{i-1}) + g_{1,i} u_{1,i} + d_{1,i} \end{aligned} \tag{11}$$

$$\begin{aligned} \dot{e}_{2,i}(t) &= -\frac{1}{\tau} [V[\rho_{d,i} - e_{1,i}] - (v_{d,i} - e_{2,i})] - \frac{1}{L_i} (v_{d,i} - e_{2,i}) \\ &\quad [v_{i-1} - (v_{d,i} - e_{2,i})] + \frac{v}{\tau L_i} \frac{[\rho_{i+1} - \rho_i]}{[\rho_i + \kappa]} - \xi_{v,i}(t) \\ &= -\frac{1}{\tau} V[\rho_{d,i} - e_{1,i}] + \frac{1}{\tau} v_{d,i} - \frac{1}{\tau} e_{2,i} - \frac{1}{L_i} (v_{d,i} - e_{2,i}) \\ &\quad v_{i-1} + \frac{1}{L_i} (v_{d,i}^2 - 2v_{d,i} e_{2,i} + e_{2,i}^2) + u_{2,i} - \xi_{v,i}(t) \\ &= \frac{1}{L_i} e_{2,i}^2 - \frac{1}{L_i} 2v_{d,i} e_{2,i} - \frac{1}{\tau} e_{2,i} + \frac{1}{L_i} v_{d,i}^2 + \frac{1}{\tau} v_{d,i} \\ &\quad - \frac{1}{\tau} V[\rho_{d,i} - e_{1,i}] - \frac{1}{L_i} (v_{d,i} - e_{2,i}) v_{i-1} + u_{2,i} - \xi_{v,i}(t) \\ &= f_{2,i}(e_{1,i}, e_{2,i}, \rho_{d,i}, v_{d,i}, v_{i-1}) + g_{2,i} u_{2,i} + d_{2,i} \end{aligned} \tag{12}$$

where  $u_{1,i}$  is the ramp input,  $u_{2,i}$  is the speed input, and the remaining terms are

$$\begin{aligned} f_{1,i} &= \frac{1}{L_i} e_{1,i} e_{2,i} - \frac{1}{L_i} v_{d,i} e_{1,i} - \frac{1}{L_i} \rho_{d,i} e_{2,i} + \frac{1}{L_i} \rho_{d,i} v_{d,i} - \frac{1}{L_i} q_{i-1} \\ f_{2,i} &= \frac{1}{L_i} e_{2,i}^2 - \frac{1}{L_i} 2v_{d,i} e_{2,i} - \frac{1}{\tau} e_{2,i} + \frac{1}{L_i} v_{d,i}^2 + \frac{1}{\tau} v_{d,i} - \frac{1}{\tau} V \\ &\quad \cdot [\rho_{d,i} - e_{1,i}] - \frac{1}{L_i} (v_{d,i} - e_{2,i}) v_{i-1} \\ g_{1,i} &= -\frac{1}{L_i}, \quad g_{2,i} = 1, \quad d_{1,i} = -\xi_{\rho,i}, \quad d_{2,i} = -\xi_{v,i}. \end{aligned}$$

It should be noted that the error dynamics in (11) and (12) will be used for the stability analysis and not for the simulations given later in section 4.

To achieve traffic flow control, the traffic density and speed should be regulated towards the desired density and speed. This can be achieved by making the system tracking errors  $e_{1,i}$  and  $e_{2,i}$  eventually converge to the origin in finite time. To this end, a sliding surface should be selected first and then an SMC law should be designed to make the states of the closed-loop system move towards the sliding surface in finite time.

**Remark 1:** The proposed method is basically based on the feedback linearization method, which makes it possible to decouple the coupled system in such a way that we can consider two separate decoupled systems. Therefore, irrespective of the coupled terms, the proposed controller design procedure can become valid.

Since we can use both RM and speed regulation for the traffic flow control, the error dynamics in Eqs. (11) and (12) can be considered as the separate subsystems. Thus, for the application of SMC to each subsystem, we select the following variables as sliding surfaces:

$$S_{1,i} = e_{1,i}, \quad S_{2,i} = e_{2,i}. \tag{13}$$

It should be noted that since each subsystem is of first

order, the sliding surfaces can be chosen as the tracking error variables. Of course, the finite-time convergence towards the sliding surfaces [i.e.,  $\rho_i(t) = \rho_{d,i}$ ,  $v_i(t) = v_{d,i}$ ] can be considered as the finite-time regulation of the tracking errors in this present case. Therefore, each individual controller should be constructed so that each sliding surface can be reached.

In subsection III-A, we will first assume that the upper bounds of the disturbances are available to design the SMC-based traffic control. Those upper bounds are then assumed to be unknown and estimated to design the ASMC-based traffic control in subsection III-B.

### 3.1 SMC-based traffic flow control

For the first subsystem of Eq. (11), we can select the control  $u_{1,i}$  as

$$u_{1,i} = u_{eq1,i} - g_{1,i}^{-1} D_{1,i} \operatorname{sgn}(S_{1,i}) \quad (14)$$

where  $\operatorname{sgn}(\cdot)$  is the sign function satisfying  $\operatorname{sgn}(a) = 1$  for  $a > 0$ ,  $\operatorname{sgn}(a) = -1$  for  $a < 0$ , and  $\operatorname{sgn}(a) = 0$  for  $a = 0$ . The symbol  $D_{1,i}$  is an upper bound of  $|d_{1,i}|$  satisfying  $|d_{1,i}| + \varepsilon_{1,i} \leq D_{1,i}$  for  $\varepsilon_{1,i} > 0$ , and  $u_{eq1,i}$  is an equivalent control given by

$$u_{eq1,i} = g_{1,i}^{-1} (-f_{1,i} - k_{1,i} S_{1,i}) \quad (15)$$

for  $k_{1,i} > 0$  such that the subsystem described by Eq. (11) can be stabilized for the global region.

In a similar way, the stabilizing controller for the subsystem of Eq. (12) can be expressed as

$$u_{2,i} = u_{eq2,i} - g_{2,i}^{-1} D_{2,i} \operatorname{sgn}(S_{2,i}) \quad (16)$$

where  $D_{2,i}$  is an upper bound of  $|d_{2,i}|$  satisfying  $|d_{2,i}| + \varepsilon_{2,i} \leq D_{2,i}$  for  $\varepsilon_{2,i} > 0$ , and  $u_{eq2,i}$  is an equivalent control given by

$$u_{eq2,i} = g_{2,i}^{-1} (-f_{2,i} - k_{2,i} S_{2,i}) \quad (17)$$

for  $k_{2,i} > 0$  such that the subsystem of Eq. (12) can be stabilized again. The stability of the overall SMC-based traffic flow control system can then be analyzed in accordance to the following theorem.

**Theorem 1 (SMC-based traffic flow control):** SMC control laws in Eqs. (14)-(17) can guarantee the asymptotic stability of each subsystem in Eqs. (11) and (12) such that the system trajectory in Eqs. (1)-(4) can achieve the desired density and speed tracking performance.

**Proof:** For the first subsystem in Eq. (11), consider a Lyapunov function

$$V_{1,i} = \frac{1}{2} S_{1,i}^2. \quad (18)$$

Therefore, if we select the control  $u_{1,i}$  as shown in Eq. (14), then its time derivative becomes

$$\begin{aligned} \dot{V}_{1,i} &= S_{1,i} \dot{S}_{1,i} \\ &= e_{1,i} \{ f_{1,i}(e_{1,i}, e_{2,i}, \rho_{d,i}, v_{d,i}, q_{i-1}) + g_{1,i} u_{1,i} + d_{1,i} \} \\ &= e_{1,i} \{ -k_{1,i} e_{1,i} - D_{1,i} \operatorname{sgn}(e_{1,i}) + d_{1,i} \} \\ &\leq -k_{1,i} e_{1,i}^2 - \varepsilon_{1,i} |e_{1,i}|. \end{aligned} \quad (19)$$

As a consequence, the subsystem of Eq. (11) can be stabilized for the global region in finite time. In a similar way, the stabilizing controller given by Eq. (16) for the subsystem of Eq. (12) can stabilize this subsystem in finite time again. (Q.E.D.)

### 3.2 ASMC-based traffic flow control

The stability analysis in the previous subsection shows that the SMC method can make the traffic flow system track the desired density and speed. This, however, requires knowledge of the upper bounds  $D_{1,i}$  and  $D_{2,i}$  that may not be available in practice. Thus, considering more practical situations, the parameter adaptation laws of the upper bounds of disturbances are designed as follows:

$$\dot{\hat{D}}_{1,i} = \gamma_{1,i} |e_{1,i}|, \quad \dot{\hat{D}}_{2,i} = \gamma_{2,i} |e_{2,i}| \quad (20)$$

where  $\gamma_{1,i}$  and  $\gamma_{2,i}$  are positive parameter adaptation gains at section  $i$ . Although the existence of the switching term guarantees that the tracking errors remain near zero, it is not suitable due to the chattering phenomenon. Thus, a saturation function can be used instead of a switching function. Still, sliding surfaces  $e_{1,i} = 0$  and  $e_{2,i} = 0$  cannot be guaranteed by using a saturation function, which, in turn, leads to the fact that the estimates of  $D_{1,i}$  and  $D_{2,i}$  are not guaranteed to be bounded. When  $e_{1,i}$  and  $e_{2,i}$  are not exactly zero,  $\hat{D}_{1,i}$  and  $\hat{D}_{2,i}$  may grow without bounds as it can be seen from the parameter adaptation laws in Eq. (20). Therefore, this problem should be solved by introducing the deadzone to the parameter adaptation laws as described in [30].

The deadzone tracking error is defined as

$$e_{k,i}^w = e_{k,i} - d_{k,i}^w \operatorname{sat}(e_{k,i} / d_{k,i}^w)$$

for  $k = 1, 2$ , where  $d_{k,i}^w$  is the width of the deadzone and selected here as a constant, and  $\operatorname{sat}(\cdot)$  is a saturation function satisfying  $\operatorname{sat}(a) = a$  for  $|a| < 1$  and  $\operatorname{sat}(a) = \operatorname{sgn}(a)$  for  $|a| > 1$ . In order to achieve better performance in the traffic flow system, an ASMC law based on RM and speed regulation is proposed, as in Fig. 3.

The parameter adaptation laws with deadzone are designed as

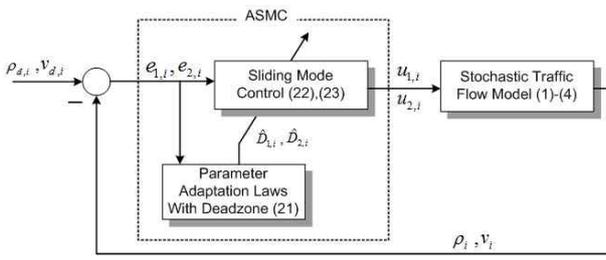


Fig. 3. Block diagram of the proposed control method

$$\begin{aligned} \dot{\hat{D}}_{1,i} &= \gamma_{1,i} |e_{1,i}| + \gamma_{1,i} d_{1,i}^w |\text{sgn}(e_{1,i}^w)| \\ \dot{\hat{D}}_{2,i} &= \gamma_{2,i} |e_{2,i}| + \gamma_{2,i} d_{2,i}^w |\text{sgn}(e_{2,i}^w)| \end{aligned} \quad (21)$$

and the ASMC laws are given by

$$u_{1,i} = u_{eq1,i} - g_{1,i}^{-1} \hat{D}_{1,i} \text{sat}(e_{1,i} / d_{1,i}^w) \quad (22)$$

$$u_{2,i} = u_{eq2,i} - g_{2,i}^{-1} \hat{D}_{2,i} \text{sat}(e_{2,i} / d_{2,i}^w). \quad (23)$$

Then, the stability of the overall ASMC-based traffic flow control system can be analyzed in accordance to the following theorem.

**Theorem 2 (ASMC-based traffic flow control):** The ASMC law with deadzone in Eqs. (21), (22), and (23) can guarantee the asymptotic stability of each subsystem in Eqs. (11) and (12), such that the system trajectory in Eqs. (1) – (4) can achieve the desired density and speed tracking performance in the sense that

$$\tilde{D}_{1,i}, \tilde{D}_{2,i}, \hat{D}_{1,i}, \hat{D}_{2,i} \in L_\infty;$$

$\hat{D}_{1,i}$  and  $\hat{D}_{2,i}$  converge to zero asymptotically;

$$\lim_{t \rightarrow \infty} |e_{1,i}(t)| \leq d_{1,i}^w, \quad \lim_{t \rightarrow \infty} |e_{2,i}(t)| \leq d_{2,i}^w.$$

**Proof:** Instead of Eq. (18), each Lyapunov function is selected as

$$V_{k,i} = \frac{1}{2} S_{k,i}^2 + \frac{1}{2\gamma_{k,i}} \tilde{D}_{k,i}^2 \quad (24)$$

where  $\tilde{D}_{k,i} = D_{k,i} - \hat{D}_{k,i}$  for  $k=1,2$ . When  $|e_{k,i}| \leq d_{k,i}^w$ , it follows that  $e_{k,i}^w = 0$ . On the other hand, when  $|e_{k,i}| > d_{k,i}^w$ ,  $\dot{e}_{k,i} = \dot{e}_{k,i}$  holds true using  $\text{sat}(e_{k,i} / d_{k,i}^w) = \text{sgn}(e_{k,i}^w)$  and thus  $e_{k,i} = e_{k,i}^w + d_{k,i}^w \text{sgn}(e_{k,i}^w)$ . So, in the case of  $|e_{k,i}| > d_{k,i}^w$ , the time derivative of the Lyapunov function in Eq. (24) can be obtained as

$$\begin{aligned} \dot{V}_{k,i} &= S_{k,i} \dot{S}_{k,i} + \frac{1}{\gamma_{k,i}} \tilde{D}_{k,i} \dot{\tilde{D}}_{k,i} \\ &= e_{k,i} \{ f_{k,i}(e_{k,i}, e_{k,i}, \rho_{k,i}, v_{k,i}, q_{i-1}(v_{i-1})) + g_{k,i} u_{k,i} \\ &\quad + d_{k,i} \} + \frac{1}{\gamma_{k,i}} \tilde{D}_{k,i} \dot{\tilde{D}}_{k,i}. \end{aligned} \quad (25)$$

If we select the control input  $u_i$  as Eqs. (22) and (23), then the time derivative of the Lyapunov function in Eq. (25) becomes negative definite as follows.

$$\begin{aligned} \dot{V}_{k,i} &= e_{k,i} \{-k_{k,i} e_{k,i} - \hat{D}_{k,i} \text{sat}(e_{k,i} / d_{k,i}^w) + d_{k,i}\} + \frac{1}{\gamma_{k,i}} \tilde{D}_{k,i} \dot{\tilde{D}}_{k,i} \\ &\leq -k_{k,i} e_{k,i}^2 - \{e_{k,i}^w + d_{k,i}^w \text{sgn}(e_{k,i}^w)\} \tilde{D}_{k,i} \text{sgn}(e_{k,i}^w) \\ &\quad + |e_{k,i}^w + d_{k,i}^w \text{sgn}(e_{k,i}^w)| \tilde{D}_{k,i} + \frac{1}{\gamma_{k,i}} \tilde{D}_{k,i} \dot{\tilde{D}}_{k,i} \\ &\leq -k_{k,i} e_{k,i}^2 + \tilde{D}_{k,i} |e_{k,i}^w| + \tilde{D}_{k,i} d_{k,i}^w |\text{sgn}(e_{k,i}^w)| + \frac{1}{\gamma_{k,i}} \tilde{D}_{k,i} \dot{\tilde{D}}_{k,i}. \end{aligned} \quad (26)$$

Using Eq. (21) it follows that  $\dot{V}_{k,i} \leq -k_{k,i} e_{k,i}^2$ . Thus, when  $|e_{k,i}| > d_{k,i}^w$ , we have  $\dot{V}_{k,i} < 0$ . When  $|e_{k,i}| < d_{k,i}^w$ ,  $V_{k,i}$  becomes bounded by a constant. Based on these two cases, it follows that  $V_{k,i}(t)$  is bounded at all times (i.e.,  $e_{k,i}$  is bounded) and  $\tilde{D}_{k,i}$  becomes constant with  $\dot{\tilde{D}}_{k,i} = 0$  from the parameter adaptation laws in Eq. (21). Consequently, the subsystems in Eqs. (11) and (12) can satisfy the property 3). This implies that the system trajectory in Eqs. (1) – (4) can achieve the desired density and speed tracking performance.

**Remark 2:** While the fundamental diagram and data measured in real-time are necessary, which is also the case with the ALINEA [4] and iterative learning control (ILC) [19] methods as well, prior information of on-ramp and off-ramp demands required by the ILC method is not required in the proposed control method. Even when uncertainties exist in these terms, these uncertainties can be considered as part of the disturbances and can be compensated by the proposed methods. In addition, the bound of disturbance magnitude is not required unlike the case of the SMC method. On the other hand, ALINEA and ILC methods cannot compensate for the disturbances. Since ALINEA is an integral-type controller based on the previous input  $u_{1,i}(t-1)$  and error  $e_1(t)$ , it is not effective, especially when the initial density value is different from the desired density, in the sense that the transient performance becomes unsatisfactory. Moreover, it exhibits a much degraded performance when large disturbances exist in the traffic flow system. In addition, as noticed in [31], the ILC control method is based on several assumptions: (i) the solution to the output regulation problem should exist, (ii) the exogenous disturbances should be known exactly, and (iii) the traffic dynamics should repeat daily throughout the repeated iterations. Therefore, ILC may not be applicable to non-recurrent traffic congestion and abnormal traffic scenarios (e.g. traffic management in the presence of incidents).

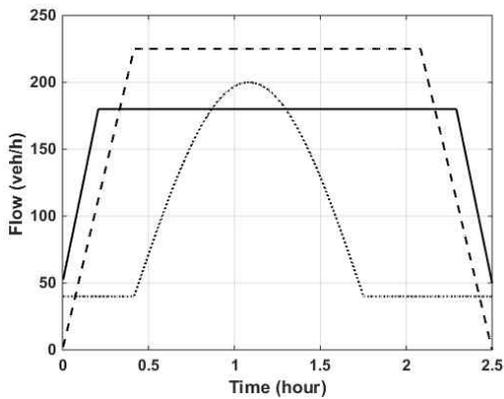
**Remark 3:** To reduce the chattering phenomena that appear in the SMC-based methods, we have employed the deadzone tracking error such that the adjustment of the deadzone can improve the robust tracking performance while eliminating the chattering, even in the presence of large disturbances in both density and speed dynamics.

Due to the introduction of the deadzone in the parameter adaptation law, the tracking error is guaranteed to be ultimately bounded and its ultimate bound can be adjusted in terms of the width of the deadzone as in the property iii) of Theorem 2.

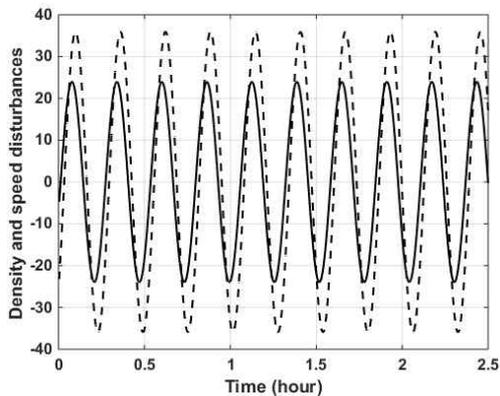
#### 4. Simulation Results

In this section, we simulate the freeway traffic system in (1)-(4) with the following setup and investigate the effectiveness and the robustness of the proposed ASMC method. In all cases, the parameters in the simulations are set to be the same as those in [19]. In this simulation, a long link of single lane freeway is divided into 12 sections and the length of each section is 500 m. The traffic volume entering section 1 is 1,500 veh/h for the entire process. For each section, the desired density is set to be  $\rho_d = 30$  veh/km/lane, the theoretical critical density is  $\rho_{cr} = 36.75$  veh/km/lane,  $v_{free} = 80$  km/h,  $\rho_{jam} = 80$  veh/k m/lane,  $l = 1.8$ ,  $m = 1.7$ ,  $\kappa = 13$  veh/km,  $\tau = 0.01$ ,  $\nu = 35$  km, and the sampling time interval is  $T = 0.00417$  h. There are two on-ramps and one off-ramp within these sections. The

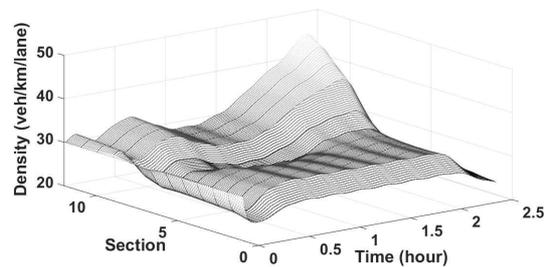
on-ramps are in sections 2 and 9 and the off-ramp is in section 7. The traffic demand pattern (on-ramp) and outflow pattern (off-ramp) in Fig. 4(a) are set to simulate the traffic situation during rush hours. Density and speed disturbances in Eqs. (1) and (2) are assumed to be sinusoids with respective amplitudes of 25 and 36, considering the traffic system's nonlinear and real-time characteristics, as shown in Fig. 4(b). In order to fully test the effect of the proposed control method, simulations were conducted under different initial positions. At the same time, simulation results were compared to those of the popular method ALINEA [32] and feedback linearization control (FLC) method [27] which is obtained by setting the signum term in (14) and (16) to be zero. It should be noted that the objective of using the deadzone is to eliminate the chattering phenomenon, which could be seen in the simulation results. However, we have omitted the detailed results for brevity. Instead, for more clearer comparison study, we have used FLC method in the following results. The initial density and speed are set to two different cases



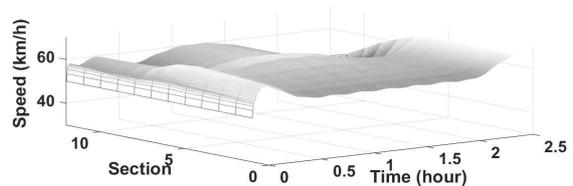
(a) Unknown traffic demands in on-ramps 2 and 9 and unknown outflow in off-ramp 7



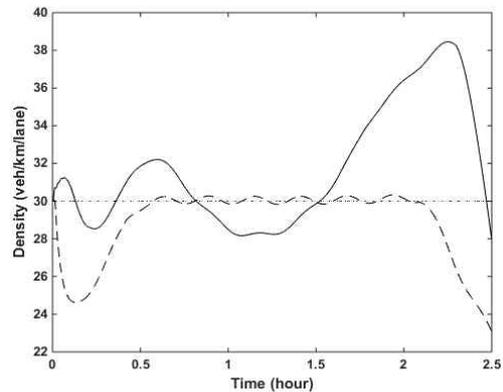
(b) Density and speed disturbances (solid: density disturbance, dashed: speed disturbance)



(a) Density profile without control



(b) Speed profile without control



(c) Density performance in sections 2 and 9 (dashed: section 2, solid: section 9, dotted: reference)

Fig. 4. Simulation conditions

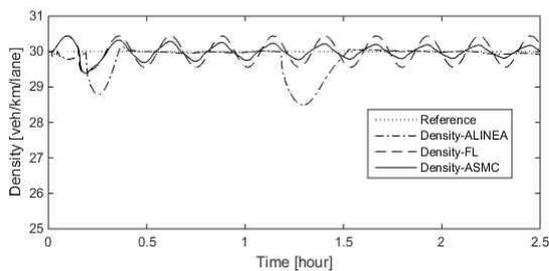
Fig. 5. Simulation results without control

(i.e.,  $\rho_{0,i} = 30$ ,  $v_{0,i} = 50$  and  $\rho_{0,i} = 25$ ,  $v_{0,i} = 60$ ) for each section. Under these simulation conditions, four cases (i.e., no control, ALINEA control, FLC, proposed ASMC method) are investigated. First, the results in the no control case are shown in Fig. 5. The results using the control method are shown in Fig. 6 in the case where the initial density value is the same as the target point (i.e.,  $\rho_{0,i} = 30$ ,  $v_{0,i} = 50$ ). Fig. 7 shows the results using the control [in the case where the initial density value is different from the target point (i.e.,  $\rho_{0,i} = 25$ ,  $v_{0,i} = 60$ )].

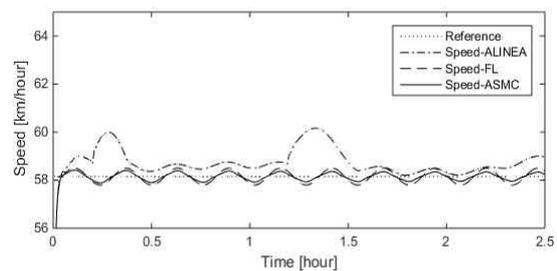
Fig. 5 shows that more vehicles appear on the freeway owing to the demand in ramps 2 and 9. Therefore, the density on the freeway becomes heavier and thus speed decreases. The density in section 9 becomes gradually larger than the critical density value, implying that traffic congestion can appear, as shown in Figs. 5(a) and 5(b). From Fig. 5(c), it is clear that the density in section 2 remains always below the desired density, which means that the influence of traffic control is not critical in the case

of section 2. Therefore, only the results in the case of section 9 are presented and discussed in detail to clearly show that the proposed traffic flow control method can prevent congestion and smooth traffic flow.

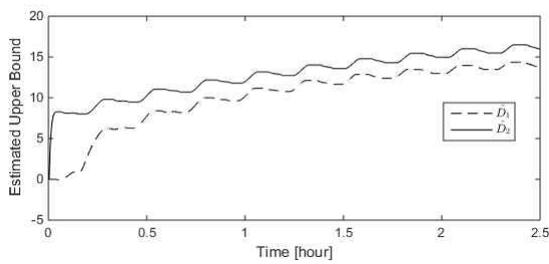
Fig. 6 shows the comparison of the performance of the proposed ASMC and that of the ALINEA and FLC in the case where the initial density value is the same as the target point. In Fig. 6(a), it is obvious that the density of section 9 is kept below the critical density ( $\rho_{cr} = 36.75$ ) so that traffic jams are avoided using the ALINEA, FLC, and ASMC methods. The performance using ASMC can significantly reduce the density drop unlike the case of ALINEA. It should be noted that the observed density drops between 0.27h and 0.34h and between 1.07h and 1.4h in Fig. 6(a) happen because of the shortage of the ramp storage (i.e., the insufficiency of the vehicles entering the mainline from on-ramps), which is inevitable. From Fig. 6(b), it is observed that the proposed method can maintain the speed tracking towards the desired speed value. Fig. 6(c)



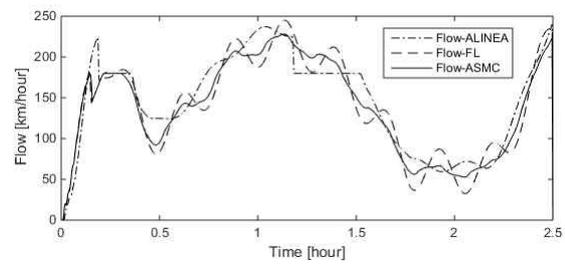
(a) Density tracking performance in section 9 (solid: ASMC, dashed: ALINEA, dash-dot: FLC, dotted: reference)



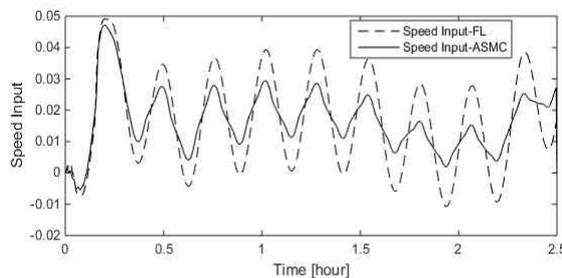
(b) Speed tracking performance in section 9 (solid: ASMC, dashed: ALINEA, dash-dot: FLC, dotted: reference)



(c) Estimates of the upper bounds of the disturbances (solid:  $\hat{D}_1$ , dashed:  $\hat{D}_2$ )



(d) Entering flow in on-ramp 9 (solid: ASMC, dash-dot: ALINEA, dashed: FLC)



(e) Speed regulation input in section 9 (solid: ASMC, dashed: FLC)

**Fig. 6.** Simulation with ramp and speed regulation in section 9 in the case that the initial density value is the same as the target point

describes the time response of the update parameters  $\hat{D}_1$  and  $\hat{D}_2$  and shows the estimation performance for the upper bounds of the disturbances. Figs. 6(d) and 6(e) show the corresponding RM and speed regulation inputs. As noticed in Remark 3, the ultimate boundedness of the tracking errors using the proposed method can be seen in these figures.

Fig. 7 shows the comparison of the performance of the proposed ASMC and that of the ALINEA and FLC in the case where the initial density value is different from the target point. In Fig. 7(a), the tracking response of the proposed ASMC method is faster than that of ALINEA and FLC. Also, an improved density and speed tracking performance can be achieved by the proposed method compared to ALINEA and FLC. The density drops observed between 0h and 0.34h and between 1.07h and 1.4h in Fig. 8(a) occur due to the inevitable shortage of the ramp storage, as mentioned previously. Fig. 7(b) shows that the speed control can maintain the speed tracking towards the desired

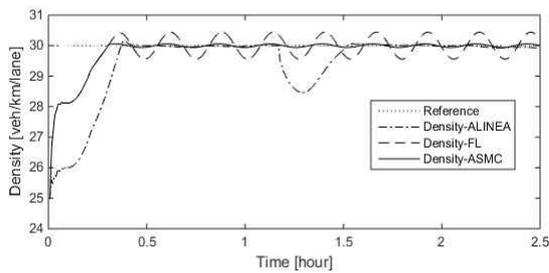
speed value. The estimation performance of the update parameters  $\hat{D}_1$  and  $\hat{D}_2$  is shown in Fig. 7(c). Figs. 7(d) and 7(e) show the corresponding RM and speed regulation inputs.

Since the density represents the traffic characteristics, the relative mean error (RME) and the root mean squared error (RMSE) [33] are selected as the performance indices, and are defined as

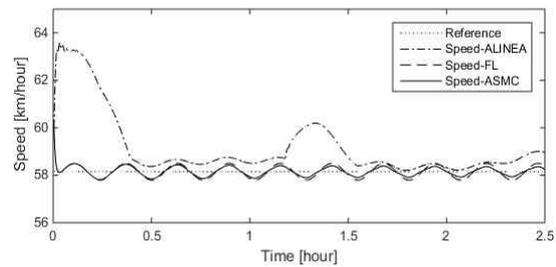
$$RME = \frac{1}{n} \sum_{i=1}^n \left| \frac{\rho_i - \rho_{d,i}}{\rho_{d,i}} \right| \quad (27)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n \left| \frac{\rho_i - \rho_{d,i}}{\rho_{d,i}} \right|^2} \quad (28)$$

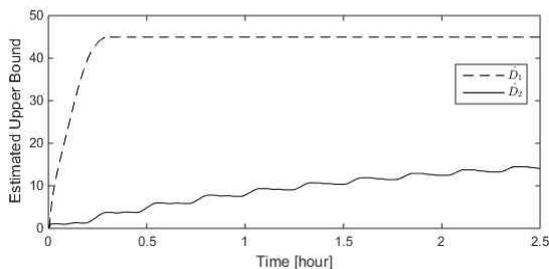
where  $\rho_i$  is the observed density value and  $\rho_{d,i}$  is the desired density value. Table 1 shows the comparison of the RME and RMSE with different control methods. The



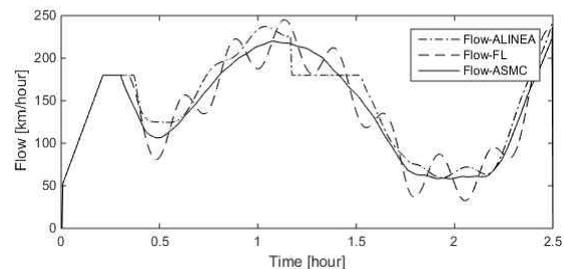
(a) Density tracking performance in section 9 (solid: ASMC, dashed: ALINEA, dotted: reference, dash-dot: FLC)



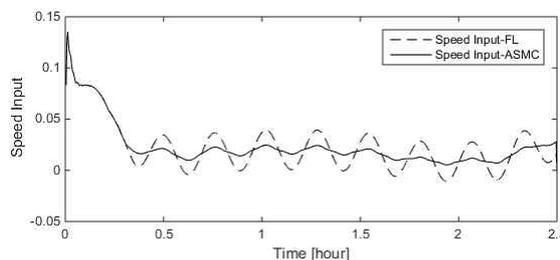
(b) Speed tracking performance in section 9 (solid: ASMC, dashed: ALINEA, dotted: reference, dash-dot: FLC)



(c) Estimates of the upper bounds of the disturbances (solid:  $\hat{D}_1$ , dashed:  $\hat{D}_2$ )



(d) Entering flow in on-ramp 9 (solid: ASMC, dash-dot: ALINEA, dashed: FLC)



(e) Speed regulation input in section 9 (solid: ASMC, dashed: FLC)

**Fig. 7.** Simulation with ramp and speed regulation in section 9 in the case that the initial density value is different from the target point

**Table 1.** Comparison of performance of several methods in terms of RME and RMSE.

Initial density	Performance indices	No control	ALINEA	FLC	ASMC
$\rho_{0,i} = 30$	RME (%)	8.72	0.70	0.96	0.52
	RMSE (%)	12.02	1.49	1.07	0.64
$\rho_{0,i} = 25$	RME (%)	9.65	1.93	1.42	0.71
	RMSE (%)	12.59	4.29	2.30	2.08

results show that the proposed control method can reduce the values of both of these indices.

From these results, it can be concluded that satisfactory results can be achieved using the proposed control method, even in the presence of ramp storage limitation and external disturbances under different initial conditions.

## 5. Conclusion

In this paper, a novel traffic flow approach based on RM and speed regulation using an ASMC method along with a deadzoned parameter adaptation law has been proposed to solve the traffic flow control problem by tracking the desired density and speed against disturbances. The stability of the overall system has been investigated through Lyapunov analyses. In the proposed control method, prior knowledge of both on-ramp and off-ramp demands is unnecessary, unlike the case of numerous previous methods. To verify the validity of the proposed method, extensive simulations for freeway traffic systems have been performed, in the presence of both density and speed disturbances under different initial conditions. It could be confirmed that the proposed control method is more effective than the ALINEA and FLC method, in the sense that both density and speed tracking performances can be maintained within satisfactory limits, even when the system disturbances are present in various initial traffic conditions. Considering that ALINEA and FLC methods cannot compensate for the disturbances, the improved performance can be naturally obtained by using the proposed method. In addition, the bounds of the disturbances are not needed in the proposed ASMC-based method based on both ramp metering and speed regulation, unlike the conventional SMC-based method which uses RM. Therefore, performance improvement, such as the maximization of the mainstream flow and the alleviation of mainline congestion, can be expected in practice by the proposed method. Still, since the adjustment of the transient control performance and the rigorous consideration of the input constraints are not easy using the proposed method, these issues can be further pursued as a future work.

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### Nomenclature

$\rho_i(t)$	Traffic density in section $i$ (veh/lane/km).
$V[\rho_i(t)]$	Density dependent equilibrium speed.
$q_i(t)$	Traffic flow leaving section $i+1$ (veh/h).
$v_i(t)$	Space mean speed (km/h).
$r_i(t)$	Off-ramp traffic volume leaving section $i$ (veh/h).
$s_i(t)$	Off-ramp traffic volume entering section $i$ (veh/h).
$L_i$	Length of the freeway in section $i$ (km).
$\xi_{\rho,i}(t)$	Disturbances acting on the density equation.
$\xi_{v,i}(t)$	Dynamic speed equation due to unknown modeling noise.
$v_f$	Free speed.
$\rho_{\max}$	Maximum possible density per lane.



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