Fuzzy PID Control by Grouping of Membership Functions of Fuzzy Antecedent Variables with Neutrosophic Set Approach and 3-D Position Tracking Control of a Robot Manipulator

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Abstract – This paper aims to design of the neutrosophic fuzzy-PID controller and it has been compared with the conventional fuzzy-PID controller for position tracking control in terms of robustness. In the neutrosophic fuzzy-PID controller, error (e) and change of error (ce) were assessed separately on two fuzzy inference systems (FISs). In this study, the designed method is different from the conventional fuzzy logic controller design, membership degrees of antecedent variables were determined by using the T(true), I(indeterminacy), and F(false) membership functions. These membership functions are grouped on the universe of discourse with the neutrosophic set approach. These methods were tested on three-dimensional (3-D) position-tracking control application of a spherical robot manipulator in the MATLAB Simulink. In all tests, reference trajectory was defined for movements of all axes of the robot manipulator. According to the results of the study, when the moment of inertia of the rotor is changed, less overshoot ratio and less oscillation are obtained in the neutrosophic fuzzy-PID controller. Thus, our suggested method is seen to be more robust than the fuzzy-PID controllers.

Keywords: Neutrosopy, Robust, Fuzzy logic controller, Grouping membership functions, Fuzzy-PID controller, 3-D position tracking control, Spherical robot manipulator.

1. Introduction

Fuzzy logic has been proposed by Zadeh [1]. It is a rule-based method and it shows susceptibility to human thinking. In fuzzy logic, linguistic variables are used such as "middle hot," "little hot," etc. A phenomenon is represented by a degree of membership which generally has a value in [0,1] range. To determine the degree of membership, membership functions (MFs) are used. Fuzzy set theory is widely used in various fields, such as image processing, signal processing, finance, medicine, defense, and robotics [2-7]. It is also widely used in the automatic control area [8-11]. A controller obtained from the procedure of design of a fuzzy logic based controller is named as fuzzy controller or fuzzy logic controller (FLC). FLC is used, particularly, in systems with uncertainty and nonlinearity [12-14].

In an FLC design, choosing the MFs type and placement of MFs of fuzzy antecedent variables on the universe of discourse is very important. Some researchers have focused on this issue. According to the results of these studies, the results of the control process are affected by the membership type and different placement [15-19]. In some other studies, researchers presented the effects of the

number of rules and output MFs on the quality of fuzzy logic control [20, 21].

After Zadeh, the fuzzy set and fuzzy logic theory have been developed by many researchers. Some of these innovations are L-fuzzy sets [22], interval-valued fuzzy sets [23-25], four-valued logic [26], intuitionistic fuzzy sets [27], interval-valued intuitionistic fuzzy sets [28], vague sets [29], and neutrosophic sets [30].

Neutrosophy was proposed by Smarandache [31]. It is an extended version of the fuzzy set/logic. Deviating from conventional fuzzy set theory, in neutrosophy theory, a phenomenon is represented by three MFs called True (T), Indeterminacy (I) and False (F) MFs. Here, T, I and F are the truth, indeterminacy and falsity value of the phenomenon, respectively. Given a data named x belonging to A set represents as $A(x)=(T, I, F)$ in the neutrosophic set theory. There is no restriction on T, I, or F. In some cases, T and F values can be equal to each other; in this case, I will have a maximum value. Furthermore, T, I and F may be dependent or independent. Therefore, a phenomenon can be interpreted focusing on neutrosophic logic rather than conventional fuzzy logic.

Controller design is a major topic in the automatic control discipline. PID or combinations of these terms (P, PI, and PD) are most commonly used in industrial process, position control, robotics, energy, and motor drives [32-36]. Although the conventional PID controller has a simple design method, it does not provide the best results for the control of unstable systems and having resonance structural

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vibration systems [37]. In literature, researchers have combined conventional PID and fuzzy set theory for obtaining more effectiveness in control results. These type controllers were called fuzzy-PID controller [38-40].

Position tracking or trajectory tracking problem is an important issue in position control applications and there are many studies in literature [41-45]. In [45], researchers have suggested a real-time position tracking system based on Field Programmable Gate Array (FPGA) and fuzzy logic. In the trajectory tracking control, a reference trajectory defines parameterized by time for the position tracking and following of this curve by the robot or robot manipulator is intended.

In the literature, there are not many studies on the use of neutrosophic logic in control applications. However, there are a few studies that have been proposed in recent years [46-53]. In the [53], researchers have combined neutrosophic logic and FLC design strategies, and they have suggested usage of neutrosophic membership functions for FLCs.

In this paper, we aimed to make an addition of neutrosophic MFs to the conventional FLC design procedure for position tracking control of the robot manipulator. For this aim, differing from conventional fuzzy logic design, we used T, I, and F neutrosophic MFs for e and ce in specified ranges of the universe of discourse for fuzzification. Therefore, these T, I, F MFs were placed in different locations (grouped) on the universe of discourse. Additionally, e and ce were evaluated separately on two fuzzy inference system (FIS) as in the [53]. Evaluation of e and ce on two FIS by using neutrosophic MFs provides a more detailed error evaluation. Because, separately evaluating of e and the ce and grouping of antecedent variables on the universe of discourse by using T, I, F MFs provides a more effective regional control. In this design, the two FIS outputs are multiplied with K_p, K_i, and K_d coefficients and the neutrosophic fuzzy-PID controller structure is designed by summing these multiplies.

As can be seen in Table 1, many studies have been done to compare the performances of PID and fuzzy-PID

controller [54-58]. According to these studies, the fuzzy-PID controllers produce good results more than conventional PID controllers, especially in terms of stability and robustness. For this reason, in this study, we realize two comparative studies for robustness between conventional fuzzy-PID controller and the neutrosophic fuzzy-PID controller. Both controllers had been adjusted the coefficients to give the minimum trajectory error and the same response time and compared in terms of robustness.

Comparative studies have been realized on a spherical robot manipulator on the MATLAB/Simulink software. In all simulation examples, a position trajectory function for all axes of the robot manipulator was used. The results for all position trajectories show that the neutrosophic fuzzy-PID controller is more robust than fuzzy-PID controllers for the different moment of inertia of the rotor (J).

This article is divided into six sections. The study is described as an outline in section one. In section two, some preliminaries are presented. In section three and section four, the methodology is presented. Section five describes how the controller's coefficients are tuned. In section six, two applications for 3-D position tracking control of a robot manipulator are realized and our suggested neutrosophic fuzzy-PID controller is compared with conventional fuzzy-PID controller. Last, results obtained from simulations are discussed.

2. Basic Information

This section presents some definitions and theorems related to the study. First, neutrosophic set theorems are given. Then, fuzzy-PID control basics are mentioned.

2.1 A Brief for Neutrosophic Set/Logic Theory

Definition 1 [48] N-norms and N-conorms for the neutrosophic logic and set are a generalization of T-norm and T-conorm from the fuzzy logic and set. In the below,

Table 1. Comparisons for PID and fuzzy-PID controllers in the literature

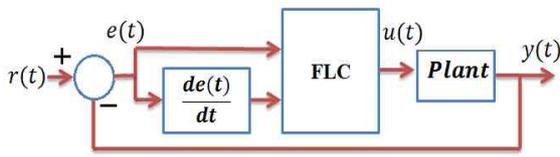| Reference number and authors	Subject of study	Comparison results and findings
H. A. Malki et all. (1997)	Designed and experimented of a fuzzy-PID controller for a flexible-joint robot arm by changing loads [54].	The fuzzy-PID controller has shown considerable tracking performance and robustness and produced overall better results than the conventional PID controller in all experiments.
C. W. Tao et all. (2005)	The comparison between the conventional PID and PID-like fuzzy controllers in terms of stability and robustness was made [55].	PID-like fuzzy controllers are more robust than the classical PID controller. And fuzzy controllers have larger robustness range.
S. Saha et all. (2012)	Using PD-type FLC and a general PD controller for position control on the different moment of inertia (J) values of the DC Servomotors [56].	Robustness test for both the controllers FLC and general PD controller has shown that FLC is more robust as compared to general PD controller.
B. Akbıyık et all. (2005)	Some analyses performed by Using PD-type FLC and a general PD controllers on various transfer functions [57].	Fuzzy-PID controllers are more robust than the classical PID controllers.
Q. Xu et all. (2014)	Trajectory tracking control of the model of a four-wheel mobile robot in different conditions by using fuzzy-PID and traditional PID controllers [58].	The fuzzy-PID controller has some advantages as against traditional PID controller. Such as rapidity, stability, anti-interference and tracking precision for trajectory tracking control of mobile robot.

"∧" operator, acting on two (standard or non-standard) subunitary sets, is a N-norm (verifying the fuzzy N-norms axioms); while the "∨" operator, also acting on two (standard or non-standard) subunitary sets, is a N-conorm (verifying the fuzzy N-conorms axioms).

A. N-norm

$$N_n: (]^-0,1^+[\times]^-0,1^+[\times]^-0, 1^+[)^2 \rightarrow]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[$$
$$N_n(x(T_1, I_1, F_1), \quad y(T_2, I_2, F_2))$$
$$=(N_nT(x, y), \quad N_nI(x, y), N_nF(x, y)) \tag{1}$$

In Eg. 1, $N_nT(x,y)$, $N_nI(x,y)$, $N_nF(x,y)$ are the truth/membership, indeterminacy and falsehood /nonmembership components, respectively.

A general example of N-norm would be this.

Let $x(T_1, I_1, F_1)$ and $y(T_2, I_2, F_2)$ be in the neutrosophic set/logic M. Then:

$$N_n(x, y) = (T_1 \wedge T_2, I_1 \vee I_2, F_1 \vee F_2) \tag{2}$$

B. N-conorm

$$N_c: (]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[)^2 \rightarrow]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[$$
$$N_c(x(T_1, I_1, F_1), \quad y(T_2, I_2, F_2))$$
$$=(N_cT(x,y), N_cI(x, y), N_cF(x, y)) \tag{3}$$

For a general example of N-conorm;

Let $x(T_1, I_1, F_1)$ and $y(T_2, I_2, F_2)$ be in the neutrosophic set/logic M. Then:

$$N_c(x, y) = (T_1 \vee T_2, I_1 \wedge I_2, F_1 \wedge F_2) \tag{4}$$

Definition 2 [31] T, I, F values are subsets of $]^-0, 1^+[$.

$$^-0 \leq \inf T + \inf I + \inf F \leq \sup T + \sup I + \sup F \leq 3^+ \tag{5}$$

Definition 3 [59] X be a universe of discourse. An element in X denoted by x. For a neutrosophic set A in X;

$T_A(x)$: Truth-membership function.
$I_A(x)$: Indeterminacy-membership function.
$F_A(x)$: Falsity-membership function.

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-,1^+[$.

$$T_A(x): X \rightarrow]0^-,1^+[$$
$$I_A(x): X \rightarrow]0^-,1^+[\tag{6}$$
$$F_A(x): X \rightarrow]0^-,1^+[$$

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$,

$$0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+ \tag{7}$$

Definition 4 [59] A single-valued neutrosophic set (SVNS) A in X. $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle: x \in X\}$.

$$T_A(x): X \rightarrow [0,1]$$
$$I_A(x): X \rightarrow [0,1] \tag{8}$$
$$F_A(x): X \rightarrow [0,1]$$

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3 \text{ for all } x \in X \tag{9}$$

Definition 5 [60] Some principal neutrosophic set operators are given below. Here, A, B and C are neutrosophic set and $x \in X$. Here, $T_A(x)$, $I_A(x)$ and $F_A(x)$ are functions of a truth membership, function of an indeterminacy membership, and function of a falsity membership of the element $x \in X$, respectively.

Intersection/AND operator:

$$T_C(x)= min (T_A(x), T_B(x))$$
$$I_C(x)= min (I_A(x), I_B(x)) \tag{10}$$
$$F_C(x)= max (F_A(x), F_B(x))$$

Union/OR operator:

$$T_C(x)= max (T_A(x), T_B(x))$$
$$I_C(x)= max (I_A(x), I_B(x)) \tag{11}$$
$$F_C(x)= min (F_A(x), F_B(x))$$

Complement/NOT operator:

$$T_A(x)= F_A(x)$$
$$I_A(x)= 1-I_A(x) \tag{12}$$
$$F_A(x)= T_A(x)$$

2.2 Fuzzy-PID Controllers

PID control is a closed loop control method commonly used in control applications, especially in industrial applications. The PID controller considers the error rate, total error, and the derivative of the error. The error rate, sum of errors, and derivative of errors are separately multiplied by coefficients indicated with K_p, K_i, and K_d respectively. The designs of PID controllers are simple and are preferred in many industrial applications. However, while PID controllers are suitable for linear systems, they are not suitable for nonlinear systems.

FLC is an alternative for situations where PID controllers are insufficient. Mamdani proposed an algorithm for fuzzy control in 1974 [61]. His algorithm is based on a linguistic control approach and human experience. Fuzzy control methodologies contain "*if – then*" rule base and MFs. The FLC design fundamentally has four main stages. FLC

Fig. 1. A general FIS structure

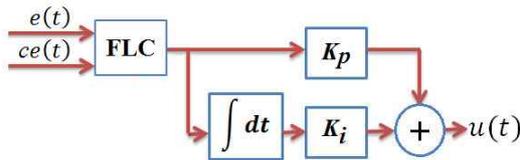**Fig. 2**. Representation of a closed loop conventional FLC block

Fig. 3. A conventional fuzzy-PID controller structure

design stages are described with a block diagram in the Fig. 1.

The first step is fuzzification. In this stage, crisp input data (mostly e and ce) are converted into fuzzy data using MFs. The second step is desig of the rule base. The rule base contains linguistic terms as "negative big," "zero," "positive little" etc. These terms, based on an expert knowledge, are placed in a specific order in a table called a rule table. Inference mechanism follows the second step. This step involves a decision making process. So, a fuzzy output value is produced. Last, the defuzzification process performs. This step converts fuzzy output values to crisp control signal. A conventional FLC is shown in Fig. 2.

Researchers combine conventional PID controller and FLC for obtain good control results more than conventional PID controllers, especially in terms of stability and robustness. In general use, the output of the FLC is the input of a PID controller. Fuzzy and PID controllers are cascaded to design a fuzzy-PID controller. A conventional fuzzy-PID controller was shown in Fig 3.

3. Methodology

Fuzzy logic contains uncertainty, and it does not clearly exhibit the uncertainty. Neutrosophic set approach contains more detailed knowledge of uncertainty (indeterminacy) than the conventional fuzzy set theory. In this study, we used medium value area of the antecedent variables on the universe of discourse as uncertainty (indeterminacy) value. This is same as with uncertainty (indeterminacy) situation in the neutrosophy. Fig. 4. and Fig. 5. summarize the suggested method.

Our proposed method is constructed based on evaluating together with the fuzzy-PID control and neutrosophic set approach. It combines the strengths of fuzzy-PID controller and neutrosophic set approach. Additionally, the method is focused on e and ce on separated two FIS unit. We used T, I, and F MFs for e and ce in specified ranges of the universe of discourse for fuzzification. Therefore, these T, I,

(a)

(b)
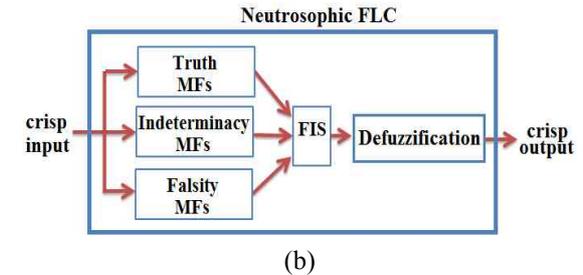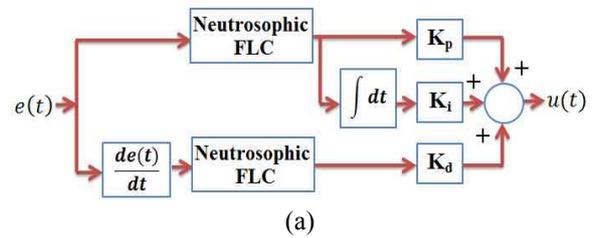
Fig. 4 (a) Suggested neutrosophic fuzzy-PID controller block diagram. (b) Internal structure of neutrosophic fuzzy-PID controller unit

(a)

(b)

(c)
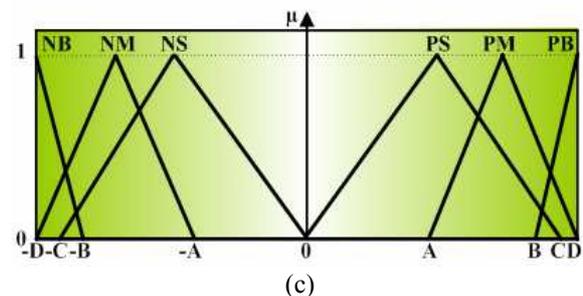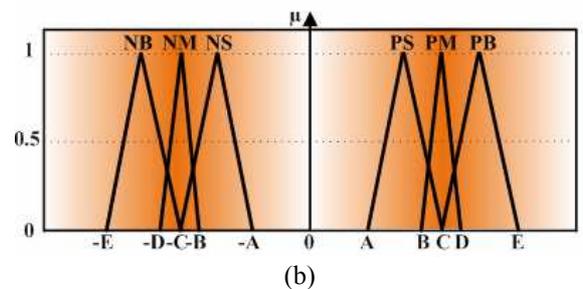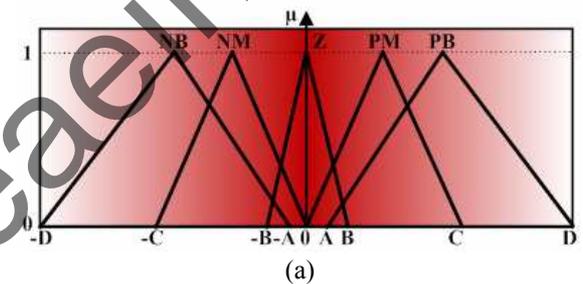
Fig. 5. Grouped neutrosophic MFs in each FIS unit. a: T MFs, b: I MFs, c: F MFs

F MFs are placed in different locations (grouped) on the universe of discourse. Accordingly, the suggested method provided a more detailed evaluation of antecedent variables

[53].

We used the same shaped (triangular for *T* and *I*, triangular and trapezoid for *F*) MFs for *e* and *ce* in specified ranges of the universe of discourse. Further, these MFs were placed in different locations (grouped) on the universe of discourse by using neutrosophic perspective.

Placement and change of the density of the *T*, *I*, *F* MFs can be seen from Fig. 5. In the dark area of the Fig. 5, MF density is high. Conversely, in the light area of the Fig 5, MF density is low. In Fig. 5, NB, NM, NS, Z, PS, PM, and PB indicate to "negative big", "negative medium", "negative small", "zero", "positive small", "positive medium", and "positive big" respectively. A, B, C, D, and E indicate the different interval of the scale of the universe of discourse.

4. Spherical Coordinates and 3-D Position Tracking Control of a Spherical Robot Manipulator

Given a *P* point in spherical coordinate is represented by (r, θ, ϕ). *r* represents to radial distance, θ represents to polar angle, and ϕ represents to azimuthal angle. A representation of the spherical robot manipulator is given in Fig. 6.

A point in the cartesian coordinate system is indicated by $P(x, y, z)$. Eq. 13 and Eq. 14 can be used for converting spherical coordinates to cartesian coordinates, and Cartesian coordinates to Spherical coordinates.

$$\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned} \tag{13}$$

$$\begin{aligned} r &= \sqrt{x^2 + y^2 + z^2} \\ \theta &= \arccos(\frac{z}{r}) \\ \varphi &= \arctan(\frac{y}{x}) \end{aligned} \tag{14}$$

In this study, we tested our suggested method on 3-D position tracking control of a spherical robot manipulator by adjusting polar angle θ, azimuthal angle ϕ , and *r*
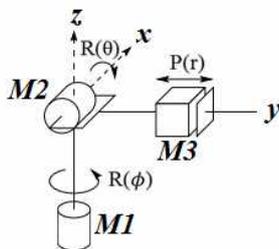
Fig. 6. A spherical robot manipulator representation [62]. *M1*: Azimuthal angle motor, *M2*: Polar angle motor, *M3*: Radial distance (elongation) motor

elongation of the robot arm in spherical coordinates. Minimum robot arm elongation was taken as 0 units and maximum robot arm elongation was taken as 10 units. The movement of the robot manipulator on the *r* direction is linear motion. *P* point represents the actual position (or end point) of the robot arm. Three permanent magnet direct current motor (PMDC) motors were used to drive θ, ϕ angles and *r* linear elongation. *M1* named motor is driving to the ϕ angle, *M2* named motor is driving to the θ angle, and *M3* named motor driving to *r* linear elongation. Three motors had the same transfer function. The transfer function of M1, M2 and M3 PMDC motors in *s* domain and their parameters are given in Eq. 15.

$$TF_{motor} = \frac{\theta(s)}{V(s)} = \frac{K}{s\big((Js + B)(Ls + R) + K^2\big)} \tag{15}$$

In Eq. 15, θ represents the motor shaft angle, and *V* represents to applied voltage to the motor. *K*, *R*, *L*, *J*, and *B* terms represent an electromotive force constant (N.m/A), and motor torque constant electric resistance (Ω), electric inductance (H), moment of inertia of the rotor (kg. m^2), motor viscous friction constant (N.m.s) respectively. The
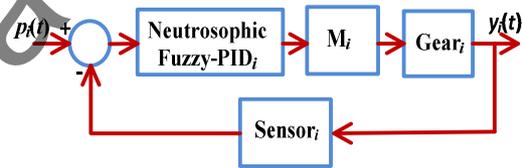

Fig. 7. Representation of 3-D position tracking control system of the robot manipulator by using neutrosophic fuzzy-PID controller (*i* =1, 2, 3 as for *r*, θ and ϕ , respectively)

Table 2. *T*, *I* and *F* MFs of *e* of ϕ angle motor

e	T	NB [-360, -180, -25.71]	NM [-205.7,- 102.9, 0]
		Z [-51.45, 0, 51.45]	PM [0, 102.9, 205.7]
		PB [25.71, 80, 360]	
	I	NB [-282.9,-231.4,-180]	NM [-205.7,-180,-154.3]
		NS [-180,-128.6,-77.16]	PS [77.16, 128.6, 180]
		PM [154.3, 180, 205.7]	PB [180, 231.4, 282.9]
	F	NB [-5e4,-5e4,-360,-313.7]	NM [-360, -257.1,-154.3]
		NS [-334.3, -180, 0]	PS [0, 180, 334.3]
		PM [154.3, 257.1, 360]	PB [313.7, 360 5e4, 5e4]

Table 3. *T*, *I* and *F* MFs of *ce* of ϕ angle motor

ce	T	NB [-90, -45, -6.429]	NM [-51.43,-25.71, 0]
		Z [-12.86, 0, 12.86]	PM [0, 25.71, 51.43]
		PB [6.429, 45, 90]	
	I	NB [-70.71,-57.86, -45]	NM [-51.43,-45, -38.57]
		NS [-45, -32.14,-19.29]	PS [19.29,2.14, 45]
		PM [38.57,45, 51.43]	PB [45, 57.86, 70.71]
	F	NB [-97.71,-90, -77.14]	NM [-90,-64.29, -38.57]
		NS [-83.57,-45, 0]	PS [0, 45, 83.57]
		PM [38.57, 64.29, 90]	PB [77.14, 90, 97.71]

weight of the robot arm and other mechanical frictions of the system were not taken into account. The parameters of the motor were taken as $K = 0.105$, $R = 2.7$, $L = 0.004$, $J = 0.0001$, and $B = 0.0000093$.

Robot arm position tracking control system is shown in Fig. 7. For θ and ϕ, the gear ratio was taken as 1/10, and for r linear axis, the gear ratio was taken as 1/100. In simulations, transfer function of position sensors has been accepted as constant 1.

Interval values of input MFs of azimuthal angle motor for e and ce in Fig. 4 are provided in Table 2 and Table 3. For the polar angle motor and radial distance motor, same shaped MFs were used. The only change was that intervals of MFs of the polar angle motor were at a rate of 1/2 of MFs of the azimuthal angle motor. Intervals of MFs of the radial distance motor are rate of 1/36 of MF of the azimuthal angle motor.

Minumum and maximum values of the universe of discourse for all axes are given in Table 4.

The output MFs of each neutrosophic fuzzy-PID controller units can be seen in Fig. 7. For each neutrosophic fuzzy-PID controller, same shaped output MFs (triangular for T and I, triangular and trapezoid for F) was used.

Interval values of output MFs for e and ce are given in Table 5.

The rule base table is provided in Table 6. Same rule base for both e and ce was used for all axes of robot manipulator. VD, D, LD, Z, LI, I, VI represents "very decrease", "decrease", "little decrease", "zero", "little increase", "increase", and "very increase," respectively.

Table 4. Minimum and maximum values of universe of discourses (UDs) for all axes**.**

Axes	min. max. values of UDs	
	e	ce
r	-10, 10	-2.5, 2.5
θ	-180, 180	-45, 45
ϕ	-360, 360	-90, 90

Table 5. Output MFs of e and ce for all axes of robot manipulator

	VD [-5e4, -5e4, -1, -0.6657]	D [-.8417, -0.4989, -0.1143]
u	LD [-0.2, -0.1143, 0]	Z [-0.04167, 0, 0.04167]
	LI [0, 0.1143, 0.2]	I [0.1143, 0.4989, 0.8417]
	VI [0.6657 1, 5e4 , 5e4]	
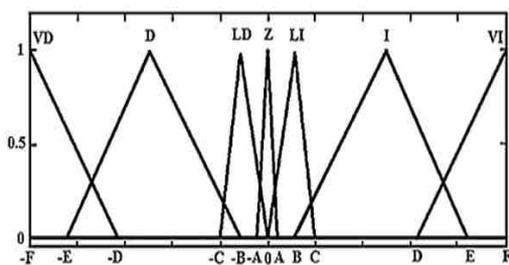
Fig. 7. The output MFs of each axes

e_T, e_I and e_F represent to MFs. As can be seen from Table 6, only 20 rules have been used. Features of the FIS are given below.

FIS type: "*mamdani*"
Rule connections: "*or*"
For "or" method: "*max*"
For defuzzification: "*centroid*"

Used rules in the suggested method are given below.

Table 6. Rule base of neutrosophic FIS

R1. If e_T is Z or e_F is PS then output1 is Z
R2. If e_T is Z or e_F is NS then output1 is Z
R3. If e_T is PM or e_F is PS then output1 is LI
R4. If e_T is NM or e_F is NS then output1 is LD
R5. If e_T is PM or e_I is PS or e_F is PS then output1 is LI
R6. If e_T is NM or e_I is NS or e_F is NS then output1 is LD
R7. If e_T is PB or e_I is PS or e_F is PS then output1 is LI
R8. If e_T is NB or e_I is NS or e_F is NS then output1 is LD
R9. If e_T is PB or e_I is PM or e_F is PS then output1 is I
R10. If e_T is NB or e_I is NM or e_F is NS then output1 is D
R11. If e_T is PB or e_I is PB or e_F is PM then output1 is I
R12. If e_T is NB or e_I is NB or e_F is NM then output1 is D
R13. If e_T is PB or e_I is PB or e_F is PM then output1 is I
R14. If e_T is NB or e_I is NB or e_F is NM then output1 is D
R15. If e_T is PB or e_F is PM then output1 is VI
R16. If e_T is NB or e_F is NM then output1 is VD
R17. If e_T is PB or e_F is PB then output1 is VI
R18. If e_T is NB or e_F is NB then output1 is VD
R19. If e_F is PB then output1 is VI
R20. If e_F is NB then output1 is VD

In all examples, we define a reference trajectory by using Eq. 16 for obtaining synchronous movements. The reason why we define a trajectory as in Eq. 16 is to provide that the robot arm reaches the desired position in the r, θ, and ϕ axes at the same time. This situation prevents the intermittent movement of the robot. For this reason, fuzzy-PID controller and neutrosophic fuzzy-PID controller were compared only in terms of robustness, while response time was not considered.

$$p(t) = p_1 + \Delta p \left(\frac{2}{1 + e^{-\left(\frac{t}{\tau}\right)^3}} - 1 \right) \qquad (16)$$

In Eq. 16, $p(t)$ is trajectory function, p_1 and p_2 are start (current) and target (last) position. Δp ($p_2 - p_1$) is different between p_2 and p_1 points. t is time and τ is time constant. τ defines to the velocity of the trajectory. For different angle values (different trajectory length), τ can be changed. Therefore, the velocity of the trajectory is adjusted.

5. Tuning of K_p, K_i, and K_d Coefficients

First, to find the coefficients of the fuzzy-PID controller and the neutrosophic fuzzy-PID controller, the middle value

of the maximum motion fields of each axis is selected as the target point ($r = 5$ cm, $\theta = 90$ and $\phi = 180$ degrees). Then, for reaching to the target point of the robot arm with the same and minimum position tracking error is updated step-by-step the both controller coefficients. These found coefficients are given in Table 7. Obtained coefficients of the controllers are used in all the application examples.

The performance indices, which are the integral of squared error (ISE), the integral of absolute error (IAE), the integral of time multiply squared error (ITSE), and the integral of time multiply absolute error (ITAE) methods, are used as the position tracking error criterion. The motor transfer function (Eq. 15), which has the same motor parameters for each axis, is used in the process of determining the controller coefficients and in the application examples. The motor parameters are taken as $K = 0.105$, $R = 2.7$, $L = 0.004$, $J = 0.0001$, and $B = 0.0000093$.

As can be seen from Table 8, according to the IAE, ITAE, ISE and ITSE performance indices, error rates obtained with fuzzy-PID controller and neutrosophic fuzzy-PID controller are close to each other. However, when the values given in Tables 7, 8 are compared for each controller, higher values of K_p, K_i, and K_d were obtained for fuzzy-PID controller to obtain approximately the same error rates. Obviously, increasing of PID coefficients cause to overshoot and oscillation in the controlled same system. This is also an indication that the fuzzy-PID controller will be more negatively sensitive to J changing. For this reason, the fuzzy-PID controller will be less robust than the neutrosophic fuzzy-PID controller.

As shown in Table 7, K_d value is not used for the conventional fuzzy-PID controller, because error change was already evaluated in the FIS unit of the conventional fuzzy-PID controller, as can be seen from Fig. 3. The used MFs and rules in the conventional fuzzy-PID controller for the ϕ axis are same as in [63]. UDs for r, θ, and ϕ axes in both controllers are the same as in Table 4.

Table 7. Obtained K_p, K_i, and K_d values from tuning process for controllers on r, θ, and ϕ axes.

Controller Type	r			θ			ϕ		
	Kp	Ki	Kd	Kp	Ki	Kd	Kp	Ki	Kd
Fuzzy PID	10	147.45	-	100	4700	-	350	18790	-
Neutrosophic fuzzy PID	3	13.13	1	65	409.3	10	140	1698	20

Table 8. Obtained performance indices from tuning process for controllers on r, θ, and ϕ axes.

Controller Type	Fuzzy PID			Neutrosophic fuzzy PID		
	r	θ	ϕ	r	θ	ϕ
IAE	25.01	25.03	25.01	25.00	25.03	25.01
ITAE	54.61	53.49	53.38	54.77	53.30	51.96
ISE	0.45	0.44	0.44	0.42	0.41	0.40
ITSE	1.03	0.99	0.99	0.99	0.93	0.90

6. Application Examples

The suggested method was applied on two examples. In the examples, for the robustness comparison, we were performed on the different moment of inertia (J) values of the motor model. Because the gripper of the robot manipulator (or end effector) will vary for different jobs and loads in real world applications. Additionally, robot arms are made of mechanical, electrical, pneumatic, and hydraulic components. They are composed of two, three or more axes. According to the additive effect of load changes on the axis, environmental condition changes or other time-dependent system parameters will create considerable instability and vibration condition on the system.

In application examples, Eq. 16 was used to define to the reference trajectory. In all examples, τ is taken as 2. The value of the τ can be determined according to path length and desired movement velocity. In all examples, K_p, K_i, and K_d coefficients given in Table 7 were used for the both controller. Also, Eq. 15 was used as the motor transfer function on the r, θ, and ϕ axes.

6.1 Example 1

In the following example, P1(-5.68,-6.84,1.56) and P2(4.29,0.81,2.50) are the start and target points in the Cartesian coordinates, respectively. By using Eq. 14, we obtained P1(9.03,80.023,230.32) and P2(5.03,60.26,10.75) as the spherical coordinates. We ran the proposed method and obtained results are given in Fig. 8.

The curve in Fig. 8 was plotted using only the suggested method. J is constant and it was taken as 0.0001. Two control methods will give the same results when J was taken as 0.0001. For this reason, different curves were not drawn for conventional fuzzy-PID controller.

J values were increased by 0.0001 units. The obtained results are given in Fig. 9, Fig. 10, and Fig. 11.

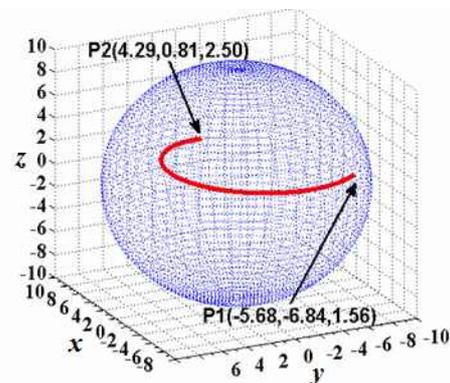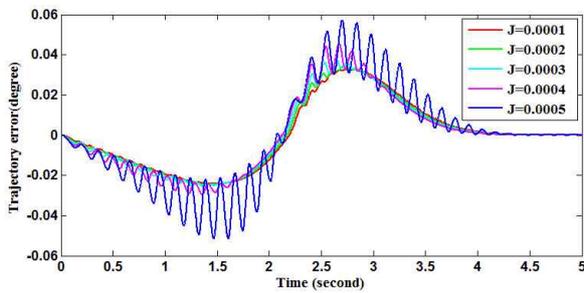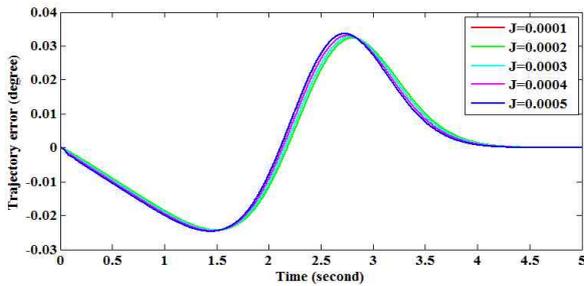As can be seen from the Fig. 9, 10 and 11 given above,

Fig. 8. Obtained robot arm trajectory result by using suggested method and representation of it on the *x-y-z* plane
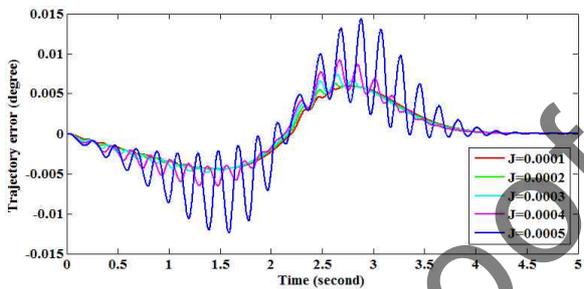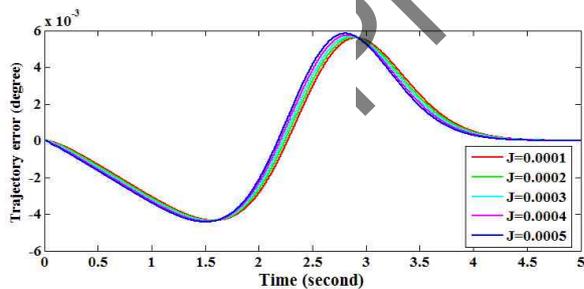
(a)

(b)

Fig. 9. Trajectory errors on the ϕ axis. (a) Conventional fuzzy-PID controller (b) Suggested method


(a)

(b)

Fig. 10. Trajectory errors on the θ axis. (a) Conventional fuzzy-PID controller (b) Suggested method

error rates within acceptable limits can be obtained with all control methods. However, oscillations occur in the r, θ and ϕ axes with varying J values. With the conventional fuzzy-PID controller, the rate of oscillation is higher when compared to the results obtained with the method proposed in the study, as seen in Fig. 9(a), Fig. 10(a), and Fig. 11(a). The results obtained by the method proposed in the study,
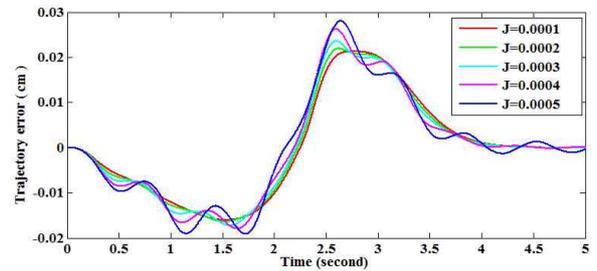
(a)
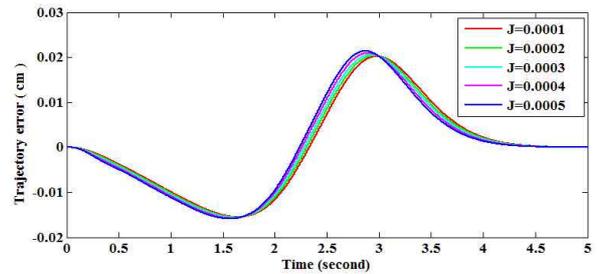
(b)

Fig. 11. Trajectory errors on the r axis. (a) Conventional (b) Suggested method
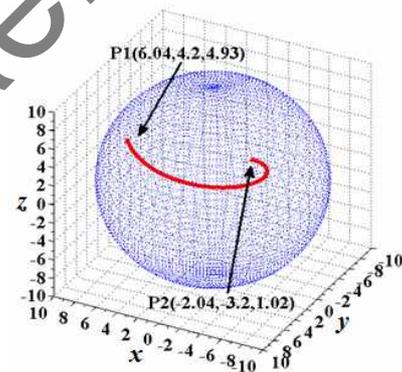
Fig. 12. Obtained robot arm trajectory result by using suggested method and representation of it on the *x-y-z* plane

the oscillation rate is much lower than the conventional fuzzy-PID controller and maintains system robust. Thus, the proposed method is less oscillated and more robust than the conventional fuzzy-PID controller (Fig. 9(b), Fig. 10(b), and Fig. 11(b)).

6.2 Example 2

In this example, P1(6.04,4.2,4.93) and P2(-2.04, -3.2, 1.02) are start and target points in the Cartesian coordinates, respectively. By using Eq. 14, we obtained P1(8.86, 56.17, 34.81) and P2(3.93, 74.96, 237.48) as the spherical coordinates. In this case, the trajectory of the endpoint of the robot arm must be a curve in 3-D space.

As in the first example, the curve in Fig. 16 was plotted using only the suggested method, here again J is constant and it was taken as 0.0001. We ran the proposed method
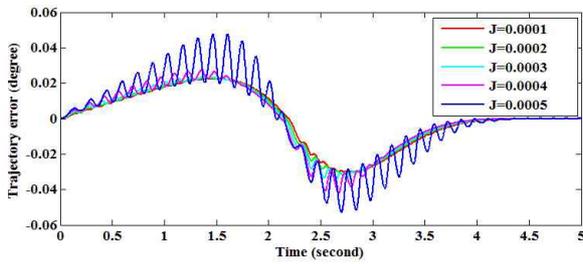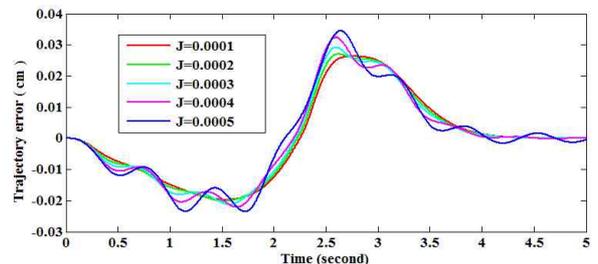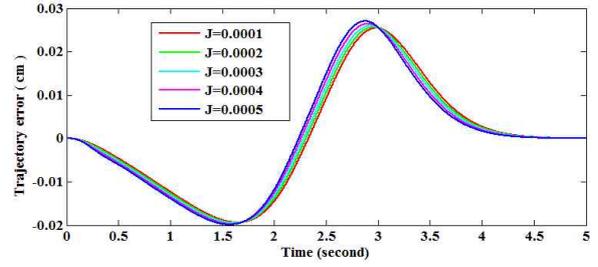
(a)
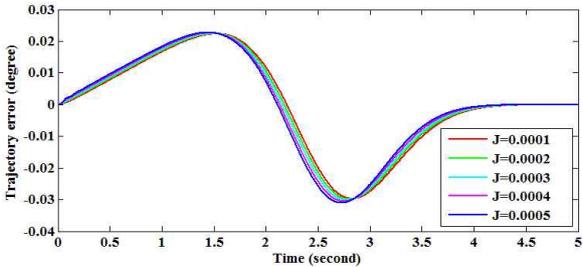
(b)

Fig. 13 Trajectory errors on the ϕ axis. (a) Conventional fuzzy-PID controller (b) Suggested method.
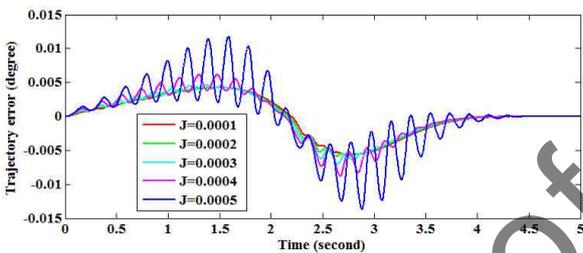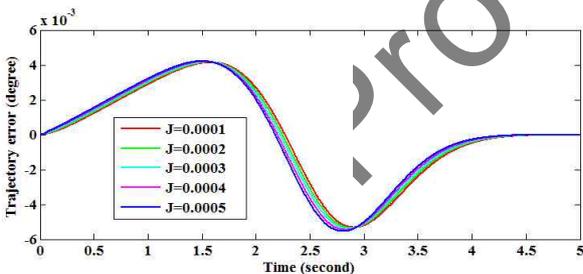

(a)

(b)

Fig. 14. Trajectory errors on the θ axis. a: Fuzzy-PID b: Suggested method

(a)

(b)

Fig. 15. Trajectory errors on the *r* axis. (a) Conventional fuzzy-PID controller (b) Suggested method

simulation results that the suggested method is more robust for load change. Conventional fuzzy-PID controller is more sensitive to load changing (*J*) and had high an oscillation for load change. The conventional fuzzy-PID controller method has greater tracking error and vibration than the suggested method.

Conclusion

In this article, we designed by the grouping MFs the neutrosophic fuzzy-PID controller for the robot manipulator trajectory control and compared to suggested method with the conventional fuzzy-PID controller. Further, we used two separate FISs for assessment of error and the derivative of the error. In all simulation examples, reference trajectory was defined for movements of the robot manipulator's axes. For the reference position trajectory, the time-dependent exponential function was used. So, the response times are equalized using the trajectory function for both controllers. By updating the coefficients of fuzzy-PID and neutrosophic fuzzy-PID controller, the position tracking error is minimized and equalized by performance indices. Robustness comparison against the moment of inertia of the rotor changes has been performed.

We tested the suggested method on the robot manipulator, and we obtained good control results. According to the obtained results, the method used in this study allows for trajectory tracking with rapid and very little tracking error. Furthermore, our suggested method had robustness for load change.

In future studies, the proposed method will be tested on

and the obtained results are shown in Fig. 12.

The *J* value increased by 0.0001 units in order to show the effects of increasing *J* values on the ϕ, θ and *r* axes. The obtained results are given in Fig. 13, Fig. 14, and Fig. 15.

As seen in Fig. 13, 14, and 15, high oscillation results were obtained with conventional fuzzy-PID and the proposed method is less oscillatory and more robust than the conventional fuzzy-PID controller. It can be seen from

the real-time robot manipulator. The conventional fuzzy-PID controller and the proposed method will be compared in load change and disturbance conditions in real time. Furthermore, neutrosophic fuzzy-PID controller design will be realized by using neutrosophic logic connectives in fuzzification, inference, and defuzzification stages and the neutrosophic fuzzy-PID controller will be examined with the respect of step function response time and compared with the fuzzy-PID controller.

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