J Electr Eng Technol.2017; 13(?): 1921-718
http://doi.org/10.???/JEET.2017.13.1.1921

ISSN(Print) 1975-0102
ISSN(Online) 2093-7423

A Novel Approved Mathematical Equation for Lightning Protection Angle

Arnon Singhasathein†, Weerapun Rungseevijitprapa and Aphibal Pruksanubal***

Abstract – During the past few decades, the lightning protection angle (α) has been proposed through several technical matters, namely graphical technique, numerical data and mathematical equation respectively. Nevertheless, these techniques are very complicated, and are difficult to utilize because they also contain several constraints practically. Hence, this paper proposes a novel equation of the lightning protection angle, which is a simple correlation, concise and easy to be implemented. Furthermore, the reliable result of this equation can confirm accuracy through comparative analysis with all previous techniques. As a result, these solutions are altogether equivalent. This novel equation can analyze the lightning protection angle of the vertical air termination system installed at the vertex of the royal pagoda in a Khema-pirataram temple which is at high risk due to lightning flashes.

Keywords: Lightning protection angle, Vertical conducting rod

1. Introduction

Lightning is a natural phenomenon, where the electric particles are discharged between cloud to cloud, cloud to earth. Nevertheless, the lightning occurrence between cloud and earth can strike an earth-bound object, which might be a human, building, the ground and so on. A lightning incident can be hazardous to life, public services, cultural places and have economic losses. Especially, tall structures are at risk from the affect of lightning. Therefore, lightning protection devices [1-8] such as an air termination (vertical conducting rod) must be utilized.

Generally, the fundamental lightning protection concept is analyzed with a vertical conducting rod. Also, it can be illustrated in equivalent mathematical models [9-16]. Moreover, these models are directly related to the lighting protection angle (α), which α is approved in the protection angle method of IEC 62305 [17-21] respectively. Normally, the limitation of α in IEC 62305 must be used for a building less than 60 meters high [19]. Not only buildings [22, 23] but also the base stations of communication equipment [6, 24, 25], solar farms [26] and so on.

Several lightning protection angle techniques have been proposed [17, 19-23]. Nonetheless, these techniques have various constraints and are difficult to be implemented in practice. The constraints of implementation, are as follows:

First, IEC 62305 standard [19, 27] presents only the results of α through a graphical technique in relation to the lightning protection angle to the object height. However, this technique does not demonstrate the mathematical equation and numerical data in practice. Thus, the graphical results are difficult to utilize when calculating the lightning protection angle.

Second, although the numerical information technique, which is directly connected between the lightning protection angle and the object height (illustrated by DEHN's document [28]), each result of the numerical information corresponds to the graphical technique of lightning protection angle of IEC 62305. Moreover, as stated in [28], the proposed results of the lightning protection angle (α) is not appropriate to be utilized in practice since it can be used only for the object height, which is an integer number, e.g., when the height is 1, 2, 3,…, 20, 30, 45 and 60 meters. Therefore, it cannot be used under practical environments, because the height of the real object may not be the integer. Furthermore, the correlation and results from this technique cannot be described through the associated mathematics. Third, the basic equation of the lightning protection angle was illustrated by Hasse & Wiesinger [29, 30]. However, it is still difficult to use due to its computational complexity and the result of α is shown in terms of radian, which needs to be converted to degrees, so it is not widely utilized.

Finally, the equal surface area technique is proposed to analyze the lightning protection angle. Generally, the equal surface area is an equivalent area (A_1=A_2) at the corresponding condition of the lightning protection angle (α) and striking distance (R) as illustrated in Fig.1 and (1). Also, the striking distance is the jumping distance between the lightning leader and streamer. Moreover, the streamer occurs from the inductive effect of the lightning leader, which is composed of 2 components, the top of the object

† Corresponding Author: Dept. of Electrical and Computer Engineering, King Mongkut's University of technology North Bangkok, Thailand. (arnon_high_voltage@hotmail.com)
* Dept. of Electrical and Computer Engineering, King Mongkut's University of technology North Bangkok, Thailand. (pruksanubal@yahoo.com)
** Dept. of Electrical Engineering, Chulalongkorn University, Thailand. (weerapun.r@chula.ac.th)
Received: May 29, 2017; Accepted: September 29, 2017

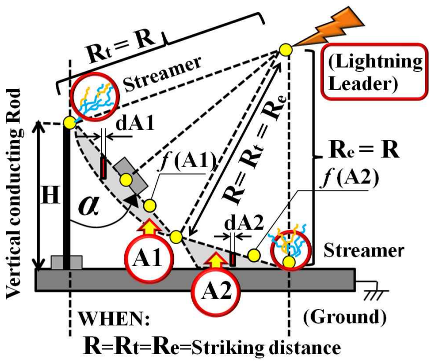


Fig. 1. Model of Equal Surface area technique

and the ground. Thus, the striking distance can be shown in the striking distance to the top of the object (R_t) and the striking distance to the ground (R_e) as presented in Fig. 1, Many researches have defined R to be equivalent to R_t and R_e ($R = R_t = R_e$) [12, 19, 27]. Therefore, the relation of (1) can be used to analyse the lightning protection angle equation, as shown in (2). However, this equation is difficult to use for the calculation because of computational complexity and the solution must be determined by numerical methods. In addition, the equal surface area ($A_1 = A_2$) is not accepted by a few researchers [31], so it is not widely utilized.

$$A_{eq} = A_1 = A_2 = \int_{AREA1} f(A_1) \bullet dA_1 = \int_{AREA2} f(A_2) \bullet dA_2 \quad (1)$$

$$2\alpha - \sin(2\alpha - 2 \cdot \arcsin(\frac{R-H}{R})) - 2\left[(\frac{A_{eq}}{R^2}) + \arcsin(\frac{R-H}{R}) \right] = 0 \quad (2)$$

where

A_{eq} is the equivalent area ($A_1 = A_2$), as shown in (1)
$f(A_1)$ is the corresponding mathematical function of A_1
$f(A_2)$ is the corresponding mathematical function of A_2
α is the result of lightning protection angle
R is the striking distance ($R = R_t = R_e$)
H is the height of vertical conducting rod

As mentioned above, the existing techniques have several constraints, which make it difficult to be practically implemented. Moreover, those constraints have not been improved. Therefore, in this paper, a novel equation to calculate the lightning protection angle from the mathematical model of a vertical conducting rod is proposed. Also, this equation can be solved by the fundamental trigonometry theorem. Essentially, it is a shorter correlation than other techniques, which is more easily utilized. Moreover, it has a prominent point which overcomes the existing techniques, as follows:

- The proposed novel equation can be used to analyze the lightning protection angle (α) with low computational complexity, gives reliable results and is easier than others to employ.
- The result of α through this novel equation can be used with realistic objects at any height, while, the result of α through the numerical information technique of DEHN [28] can only be utilized when the object height is an integer number.
- The novel equation generates a result α in terms of degree which is easy to use in practice. While, the result of the lightning protection angle equation through Hasse & Wiesinger [29] is shown α in terms of radian.
- Results of the novel equation are comparatively analyzed to confirm the accuracy with existing techniques. As a result, these are equivalent. Thus, the novel equation is appropriate to be utilized in practice.

Moreover, this paper utilizes the proposed novel equation to determine the lightning protection angle by installing it at the vertex of the royal pagoda in Khema-pirataram temple, Thailand.

The remainder of this paper is organized as follows. The novel approach to lightning protection angle equation is described in section 2. The results of the novel equation are compared with the result of the existing techniques in section 3. A practical implementation of the proposed novel equation is shown in section 4. Finally, conclusions are presented in Section 5.

2. The Analysis of a Novel Equation of Lightning Protection angle

In this section, the mathematical model and a novel equation of lightning protection angle are described. Generally, the angle of lightning protection at any level of height is implemented with a vertical conducting rod. According to Electrogeometric model (EGM) [9-16, 31-35], it can be illustrated in Fig. 2. The model of a vertical conducting rod relates to the rolling sphere method is shown [31-34]. Striking distance (R) is defined to be equivalent to the striking distance to the top of an object (R_t) and the striking distance to the ground (R_e) respectively [12, 19, 27]. Moreover, the four critical parameters, namely, the lightning protection angle (α), the striking distance (R), the lightning current (I) and the height (H) must be determined. Hence, the relationship of striking distance is given by [19].

$$R = 10 \bullet I^{0.65} \quad (3)$$

According to Fig. 2, the significant parameters for analysis of the novel equation can be determined by considering Fig. 3 and (4) to (9).

$$S = \tan(\alpha) \bullet (\sqrt{2RH}) \bullet \sin\left(\arccos(\sqrt{\frac{2R-H}{2R}}) \right) \quad (4)$$

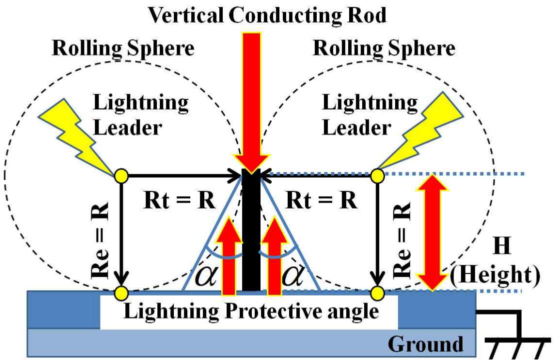


Fig. 2. EGM of vertical conducting rod

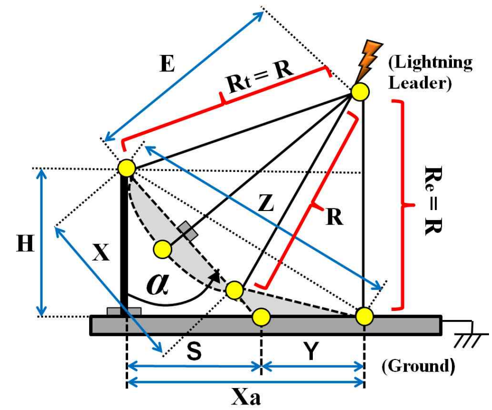


Fig. 3. Significant distances for analysis of α

$$X_a = \sqrt{2RH - H^2} \quad (5)$$

$$Y = (\sqrt{2RH - H^2}) - \left(\tan(\alpha) \bullet \sqrt{2RH} \bullet \sin(\arccos(\sqrt{\frac{2R-H}{2R}}))\right) \quad (6)$$

$$X = 2R \bullet \sin(\alpha - \arcsin(\frac{R-H}{R})) \quad (7)$$

$$E = R \bullet \cos(\alpha - \arcsin(\frac{R-H}{R})) \quad (8)$$

$$Z = (H^2 + (\sqrt{2RH - H^2})^2)^{\frac{1}{2}} \quad (9)$$

where

- R is the striking distance, as shown in (3) [19]
- H is the height of vertical conducting rod
- S is the distance of radius protection area [19, 27]
- X_a is the lateral distance [11, 35-37]
- Y is the difference distance between X_a and S
- X is the distance which has angle α degree from the height of vertical conducting rod
- E is the distance from lightning leader to be orthogonal with X distance [30]
- Z is the Hypotenuse distance as $(H^2 + X_a^2)^{1/2}$

From the relationship between (4) to (9), they are concerned with the component angles, namely β, ε, ν,

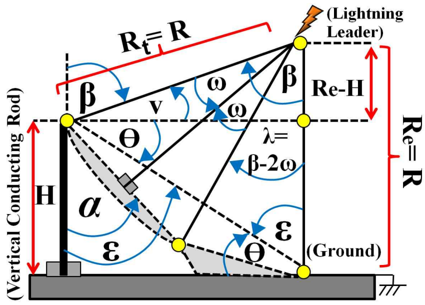


Fig. 4. Model of proposed lightning protection angle

ω, λ and θ respectively. Therefore, these are shown in Fig. 4.

As depicted in Fig. 4, the novel equation should directly relate between lightning protection angle (α) and object height (*H*). Essentially, it can also be analyzed by the trigonometry term [38-42]. Therefore, those component angles, that need to be considered, are expressed as follows (10) to (15).

$$\beta = \arccos(\frac{R-H}{R}) \quad (10)$$

$$\varepsilon = (90^{\circ} - \theta) \quad (11)$$

$$\nu = \arcsin(\frac{R-H}{R}) \quad (12)$$

$$\omega = \alpha - \arcsin(\frac{R-H}{R}) \quad (13)$$

$$\lambda = \beta - 2\omega \quad (14)$$

$$\theta = \arcsin(\sqrt{\frac{H}{2R}}) \quad (15)$$

where

- α is the lightning protection angle
- β is the angle between R_t and R_e
- ε is the angle between R_e and Z
- ν is the angle between R_t and Horizontal level
- ω is the angle between R_t and E
- λ is the difference angle between β and $2 \bullet \omega$
- θ is the angle between X_a and Z

According to Fig. 3 and 4, by analyzing (5), (9) and (15) through the Law of cosine [38-39], the relationship of the object height (*H*) can be shown as (16).

$$H = [(\sqrt{2RH})^2 + (\sqrt{2RH - H^2})^2 - 2(\sqrt{2RH} \bullet \sqrt{2RH - H^2})\cos\theta]^{\frac{1}{2}} \quad (16)$$

Referring to (16), it can be considered to update the relation of θ, which is expressed in (17).

$$\theta = 0.5 \bullet \arccos(\frac{R-H}{R}) \quad (17)$$

Moreover, the parameter ε in (11) can be modified through the relationship of (17). Thus, it can be expressed in (18).

$$\varepsilon = (90^{\circ} - \theta) = 0.5 \bullet (180^{\circ} - \arccos(\frac{R-H}{R})) \tag{18}$$

According to (7), (8), (9), (13) and (18), these are analyzed to determine the most important parameters, namely ω and α, as shown in (19) - (20), respectively.

$$\omega = \frac{\varepsilon}{2} - \frac{1}{2}(\arcsin(\frac{R-H}{R})) \tag{19}$$

$$\alpha = \omega + \arcsin(\frac{R-H}{R}) \tag{20}$$

From (19) and (20), the both relationships can be analyzed to modify the α as shown in (21).

$$\alpha = \frac{0.5 \bullet (180^{\circ} - \arccos(\frac{R-H}{R})) + \arcsin(\frac{R-H}{R})}{2} \tag{21}$$

Essentially, the relationship of (21) is proved through the trigonometry theorem [39-42]. Therefore, the novel equation of α can be obtained in the function of *R* and *H* as (22).

$$\alpha = 0.5 \bullet \left(\arctan(\frac{\sqrt{2RH - H^2}}{H}) + \arcsin(\frac{R-H}{R}) \right) \tag{22}$$

where

R is the striking distance, as shown in (3)

H is the height of vertical conducting rod

In addition, the novel Eq. (22) and the whole relationship of lightning protection angle techniques can be comparatively shown in Table 1. Moreover, the details of Table 1 can be described as follows:

First, referring to the document of DEHN [28], it shows only the numerical information about lightning protection angle. Nevertheless, it cannot illustrate the associated mathematical equation. While, this novel equation is obviously shown in the complete mathematical equation and is easy to be used in practice. Second, the result of α through numerical information from DEHN's technique can only be used on objects with a high integer number, which does not cover realistic applications. However, the novel Eq. (22) can deliver the result at any height of object.

Third, this novel equation is simple and is in an easy form with low constraints. Thus, it is easier to be utilized than the equation of Hasse & Wiesinger [29] and the equal surface area technique ($A_1 = A_2$) of (2).

Fourth, all results of α through the novel equation can be demonstrated in degree units. The equation of Hasse & Wiesinger is only shown in radian units. The result of Hasse and Wiesinger's equation must be multiplied with a converted factor ($180^{\circ}/\pi$) as shown in Table 1. Thus, it is computational complexity.

Table 1. The comparison of mathematical equations of lightning protection angle [28, 29]

Techniques	References	Mathematical Equations of Lightning Protection Angle
DEHN	REF [28]	No Mathematical Equation & Results from DEHN's information. The total height is considered and adjusted to be integer.
Hasse & Wiesinger	REF [29]	$\alpha = \frac{180}{\pi} \bullet \arctan\left[\left(\frac{1}{H} + \frac{R}{H^2}\right)\sqrt{2RH - H^2} - \left(\left(\frac{R}{H}\right)^2 \bullet \arccos(\frac{R-H}{R})\right)\right]$
Equal Surface area, (A_1=A_2)	Eq. (2)	$2\alpha - \sin(2\alpha - 2 \bullet \arcsin(\frac{R-H}{R})) - 2\left[(\frac{A_{eq}}{R^2}) + \arcsin(\frac{R-H}{R})\right] = 0$
Novel Equation	Eq. (22)	$\alpha = 0.5 \bullet \left(\arctan(\frac{\sqrt{2RH - H^2}}{H}) + \arcsin(\frac{R-H}{R})\right)$

As previously mentioned above, the prominent points of this novel equation are simplistic, shorter terms than others, lower constraint and is easy to use in actual practice. Nevertheless, this equation must be evaluated focusing on result accuracy. Hence, the next section shows the comparative analysis between the solution of the novel equation and existing techniques.

3. Simulation Results

In this section, the novel equation result is compared with three other techniques, DEHN, Hasse & Wiesinger and the equal surface area ($A_1 = A_2$). These results relate to the object height as illustrated in Fig. 5 to 8. As a result, the novel equation (as shown by the red line) has a corresponding solution with the existing techniques.

Essentially, the important condition in this section are simulated at 4 levels of lightning protection based on the IEC standard 62305 [17-19] as shown in Table 2. Namely, the minimum lightning current of 3, 5, 10, 16 kA and the striking distance of 20, 30, 45 and 60 meters respectively.

This simulation focuses on object height of more than 2 meters. This is because the IEC 62305 [19] defines the result of α at object height of less than 2 meters ($H < 2$ meters) to be equal to the results of α at object height of 2 meters ($H = 2$ meters).

As depicted in Fig. 5 to 8, the comparison results between a novel equation and existing techniques at 4 levels were obtained. The results of all techniques corresponded with each other.

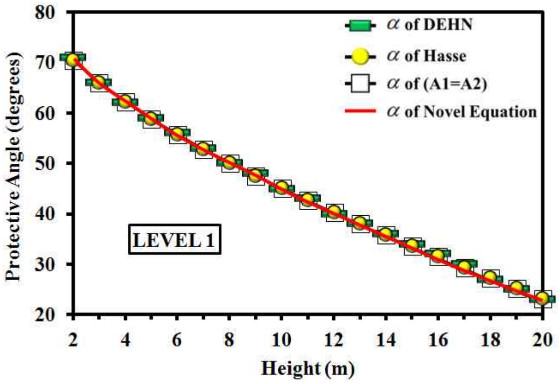


Fig. 5. Results of lightning protection angle at level 1

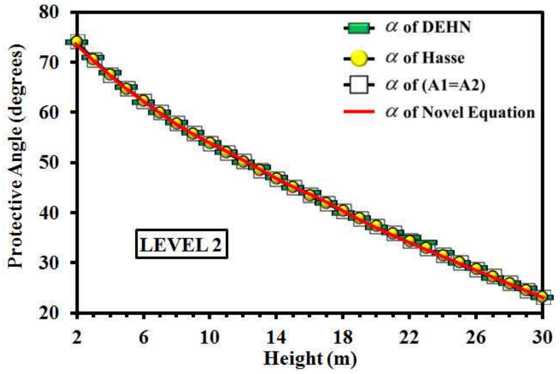


Fig. 6. Results of lightning protection angle at level 2

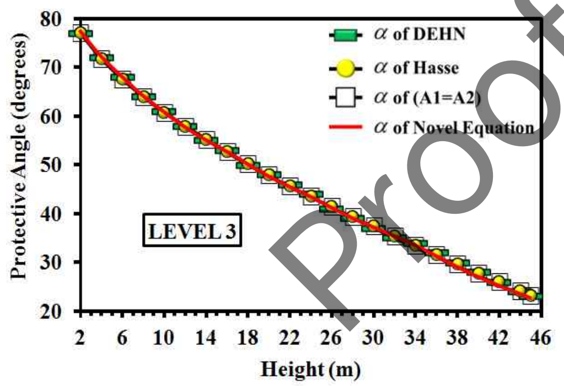


Fig. 7. Results of lightning protection angle at level 3

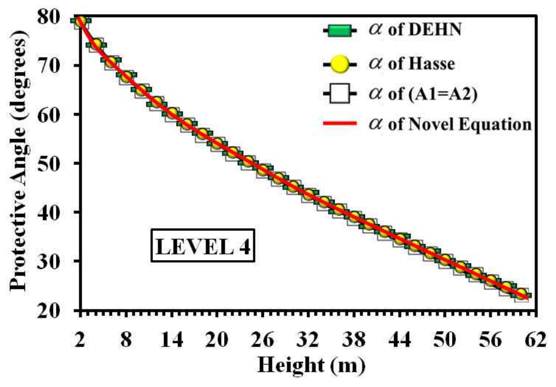


Fig. 8. Results of lightning protection angle at level 4

Nevertheless, the novel equation is the most reliable, because various conditions of error, which occur in other techniques, can be eliminated. The error of existing techniques can be explained as follows:

- DEHN's technique demonstrates the numerical data of α only, which cannot obviously show the mathematical equation to confirm an accurate result. Hence, the correctness of this technique is unclear.
- The result of α through Hasse & Wiesinger's equation is illustrated in radian units. However, it must be converted into degree units through multiplication with ($180^{\circ}/\pi$), which causes α to contain many decimal digits or non-repeating decimal numbers [43]. Also, the number must be rounded off. Therefore, α delivers the round-off error [42-44].
- The result of the equal surface area technique ($A_1 = A_2$) is determined through the numerical method such as the Newton – Raphson method [44] which directly relates to the iteration number. The iteration number must be appropriately defined to the condition of exact solution [42-44]. Thus, α can deliver error when iteration number is an unsuitable (error due to iteration) [44]. Moreover, the equal surface area technique was proved by A_1 and A_2 areas balance each other which this assumption is not widely accepted [31].

Nonetheless, the novel equation can avoid errors as shown above, because the novel equation outperforms the existing techniques, (i.e., clear relationship, non-converting to degree units, easy to analytically find exact result), as mentioned in Table 1. Also, it was directly proved through the assumption of the striking distance (R) according to IEC 62305-3 [19]. Therefore, the result of the novel equation delivers higher accuracy, precision, and reliability.

Furthermore, the comparison of error between the novel equation and other techniques can be illustrated, such as the maximum absolute error, the minimum absolute error and the average absolute error as shown in Fig. 9 to 11.

Fig. 9 shows the maximum absolute error between the novel equation and two techniques, Hasse & Wiesinger and the equal surface area (A_1=A_2), at 0.7291 degrees.

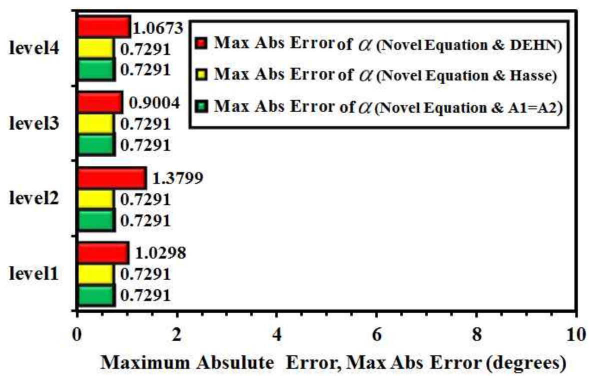


Fig. 9. Maximum absolute error of α through the novel equation with existing techniques

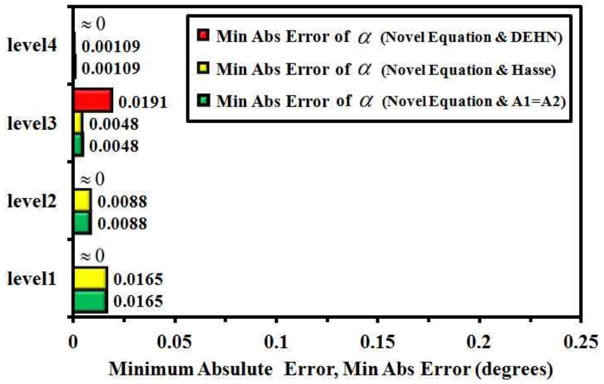


Fig. 10. Minimum absolute error of α through the novel equation with existing techniques

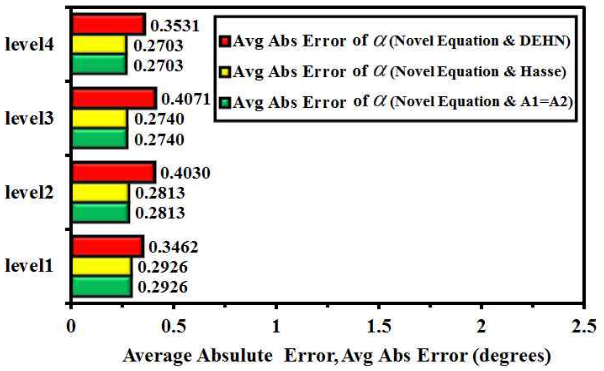


Fig. 11. Average absolute error of α through the novel equation with existing techniques

Moreover, the highest error is 1.3799 degrees, which occurs through comparative with the result of DEHN respectively. These results are insignificant, due to the fact that the solution of each technique is not completely equivalent. The minimum absolute error as shown in Fig. 10, is very tiny and nearly zero which can be neglected. Furthermore, Fig. 11 shows the average absolute error which has very low tolerance of no more than 0.4071 degrees. As described earlier, these errors are diminutive which do not influence the reliability of the novel equation when implemented. Also, the novel equation can eliminate the weaknesses of other techniques. Therefore, the novel equation is the most appropriate to be utilized in actual implementation.

The novel equation is proposed in this paper as (16), which can be utilized to design the lightning protection angle. The next section shows the application of the novel equation.

4. Practical Implementation

In this section, the lightning protection angle (α) has been implemented by using the novel equation. A vertical conducting rod is installed at the vertex of the royal pagoda

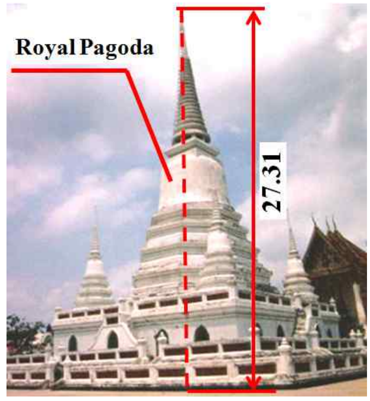


Fig. 12. The royal pagoda in Khema-pirataram temple

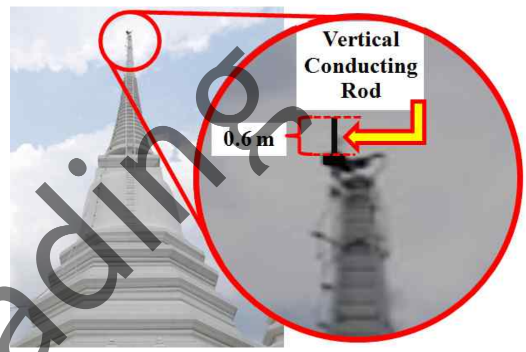


Fig. 13. The actual installation of a vertical conducting rod at the vertex of the royal pagoda

Table 2. The condition of lightning protection level based on the IEC 62305 [17-19]

LEVEL	Minimum Lightning Current. I_{min} (kA)	Striking Distance IEC Standard. R(meters)	Height (meters)
1	3	20	$(1 \le H \le 20)$
2	5	30	$(1 \le H \le 30)$
3	10	45	$(1 \le H \le 45)$
4	16	60	$(1 \le H \le 60)$

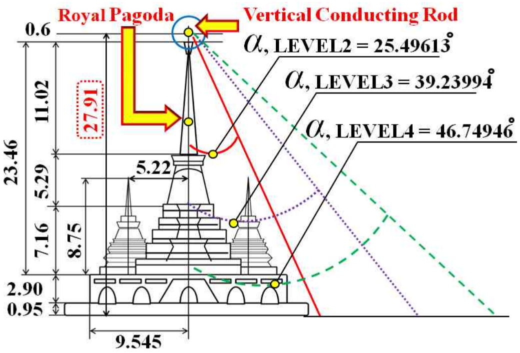
in Khema-pirataram temple, Thailand. Khema-pirataram is one of the oldest temples in Thailand with an age of more than 162 years. At the temple, there are Buddhist relics, Śarīra, placed in the royal pagoda [45] as shown in Fig. 12.

According to the investigation, it was found that the height of the royal pagoda is 27.31 meters and is frequently struck by lightning, because the structure of a royal pagoda is tall and sharp. Moreover, it is located outdoors which makes it high risk to lightning strikes. Therefore, the vertical conducting rod of 0.6 meters, needs to be installed at the vertex of royal pagoda. The total height is 27.91 meters as illustrated in Fig. 13 and Fig. 14(a). The analysis of the lightning protection angle must consider the constraints below.

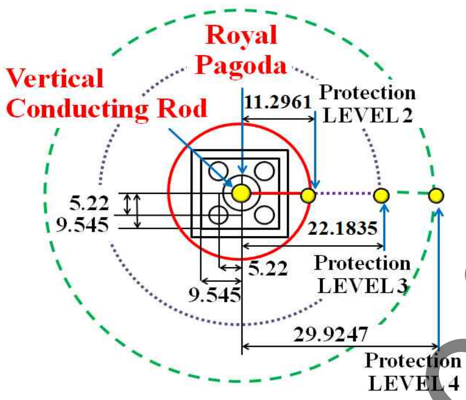
- The total height in the case study is not an integer number. Thus, DEHN's technique cannot show the solution of protective angle. In addition, the result

Table 3. Results of lightning protection angle

Technique	Reference	Height (meters)	Lightning Protection Angle (degrees)		
			Level 2	Level 3	Level 4
Novel Equation	Equation 22.	27.91	25.4961	39.2399	46.7494



(a) Front view



(b) Top view

Fig. 14. The effectiveness of lightning protection angle

through Hasse & Wiesinger is shown in radian units which are not appropriate for use.

- The equal surface area technique should not be utilized practically due to it is computational complexity.

However, the novel Eq. (22) is more beneficial than other existing techniques. Essentially, the total height of this case study is more than 20 meters, which the IEC 62305 [17, 19] recommends it to be used at level 2-4, as mentioned in Table 2. In addition, the 0.6 meters vertical conducting rod has effectiveness for lightning protection of the royal pagoda, as shown in Fig. 14(a), Fig. 14(b) and Table 3.

From the details in this section, a novel equation was presented. It has advantages when implemented such as it is easy to use, has low constraints and delivers reliable results. Moreover, it can save time when calculating the protective angle of a vertical conducting rod where a prominent point is exposed.

From the details in this section, a novel equation was presented. It has advantages when implemented such as it is easy to use, has low constraints and delivers reliable results. Moreover, it can save time when calculating the protective angle of a vertical conducting rod where a prominent point is exposed.

5. Conclusions

This paper proposed a novel equation to calculate the lightning protection angle. Also, it was proved through the fundamentals of the trigonometry theorem and the striking distance according to IEC 62305-3. Essentially, this novel equation of α is a simple equation and has no complexity of constraints, because it outperforms the existing techniques. Therefore, it can be easily utilized for implementation, and has more advantages over existing techniques. The result of the novel equation delivers high accuracy and reliablility.

Furthermore, since objects such as, tall buildings, base stations for communication, petroleum and chemical tanks, solar farms, cultural heritage buildings and so on, are at risk of being struck by lightning, vertical conducting rods can protect against the lightning effects if installed correctly. Therefore, the result of the lightning protection angle is necessary, and can be easily analyzed through this novel equation.

Finally, this paper presented a case study of estimating lightning protection angle, a vertical rod was installed on the vertex of the royal pagoda. In conclusion, this novel equation is the most appropriately utilized method due to no complexity of constraints when implemented, while existing techniques have various limitations.

References

[1] R.H. Golde, *Lightning Protection.*, London, UK: Edward Arnold, 1973, pp. 68-83.

[2] J.L. Marshall, *Lightning Protection.*, USA: John Wiley & Sons, 1973, pp. 145-164.

[3] R. H. Lee, "Protection zone for buildings against lightning strokes using transmission line protection practice" *IEEE Transactions on Industry Applications.*, vol. 14, no. 6, pp. 465-469, Nov.1978.

[4] P.Hasse and DEHN+SÖHNE, "History of Lightning Protection", *presented at IEC TC 81 Memorial Meeting*, *Institute of Electrical Installation Engineering of Japan*. Japan, 1988, pp. 7-20.

[5] Technical Committee of the British standard GEL/81: Protection against Lightning, *Code of practice for protection of structures against lightning*, British standard 6651., 1999.

[6] IEC Technical Committee 81: Lightning protection, *Protection of Structures Against Lightning Part 1-2 General* Principles-*Guild B-Design installation maintenance and inspection of lightning protection*

systems., International Standard IEC 61024-1-2., 1998-05.

[7] M.L.Henshaw, *Protection against lightning A UK guide to the practical application of BS EN 62305*. London ,UK: British Standards Institution 2007.

[8] G. Parise and L. Martrano, "The lightning protection system of a complex of structure (LPCS)" *IEEE Systems Technical Conference on Industrial and Commercial Power 2005.* New York, USA, 2005.

[9] T. Horvath, *Computation of Lightning Protection.* New York, USA: John Wiley & Sons, 1928, pp. 33-45.

[10] M.A.Sargent, "The frequency distribution of current magnitudes of lightning strokes to tall structures" *IEEE Transactions on Power Apparatus and Systems.*, vol. 95, no. 5, pp. 2224-2229, Dec. 1971.

[11] R.S.Velazquez, V.G. Gerez, D.M.Mukhedkar and Y.G.Gervais, "Probabilistic calculations of lightning protection for tall buildings" *IEEE Transactions on Industry Applications.,* vol. 18, no. 3, pp. 252-259, Nov.1978.

[12] A.M. Mousa, K.D. Rivastava, "The implication of the eletrogeometic model regarding effect of height of structure on the median amplitude of collected lightning strokes" *IEEE Transactions on Power Delivery.,* vol. 4, no. 2, pp. 1450-1460, May.1988.

[13] P.Petterson, "A Unified Probabilistic Theory of the Incidence of Direct and Indirect Lightning Strikes" *IEEE* Transactions *on Power Delivery.*, vol. 6, no. 3, pp. 1303-1310, July.1991.

[14] F.A.M. Rizk, "*Modeling of lightning incidence to tall structures part 1: theory*" *IEEE Transactions on Power Delivery.*, vol. 9, no. 1, pp. 162-171, Jan.1994.

[15] A. Kuchler, *Hochspannungstechnik Grunglagen*-Technologie-*Anwendungen*. Berlin, Germany: Springer, 2005, pp. 497-498.

[16] B.Brusso "The eletrogeometric model of the rolling sphere method" *IEEE Industry Applications Magazine.,* vol. 22, no. 2, pp. 7-70 May. 2016.

[17] IEC Technical Committee 81: Lightning protection*, Protection Against Lightning Part 1 General Principles*., International Standard IEC 62305-1., 2010-12.

[18] IEC Technical Committee 81: Lightning protection*, Protection Against Lightning Part 2 Risk Management*., International Standard IEC 62305-2., 2010-12.

[19] IEC Technical Committee 81: Lightning protection*, Protection Against Lightning Part 3 Physical Damage to Structures and Life Hazard*., International Standard IEC 62305-3., 2010-12.

[20] NFPA Technical Committee on Lightning Protection, *Standard for installation of lightning protection systems*., National Fire Protection Association (NFPA) 780., 2011 Edition.

[21] G.Parise, L.Martrano and M.Lucheroni, "Level, class, and prospected safety performance of a lightning protection system for a complex of structure (LPCS)" *IEEE Transactions on Industry Applications.,* vol. 46, no. 5, pp. 2106-2110, Sep.2010.

[22] A.M.G.Minto, "Lightning protection of building" *Wiring Installations and* Supplies (*published by the IET journal & magazines*), vol. 1983, no. 16, pp. 5-10, April. 1983.

[23] H. Tettnang and Workgroup, *Tabellenbuch Elekrotechnik, 22nd edition, 2007.,* Germany: Europa-Lehrmittel, 2007, pp.238-239.

[24] Committees of the IEEE Standards, *IEEE Guide for the* Protection *of Communication Installations from Lightning Effects*., IEEE Std 1692-2011.

[25] P.Hasse and J.Wiesinger, *EMV Blitz-schutzzonn Komzept*, Munchen, Germany: Pflaum Verlag 1993.

[26] C.A. Charalambous, N.D. Kokkinos, and N. Christofides, "External Lightning Protection and Grounding in Large-Scale Photovoltaic Applications" *IEEE Transactions on Electromagnetic Compatibility.,* vol. 56, no. 2, pp. 427-434, Aug. 2013.

[27] Thai Standard, *Protection Against Lightning Part 3 Physical Damage to Structures and Life Hazard (The Engineering Institute of Thailand under H.M. The King's Patronage, EIT Standard, 2007-53)*. Bangkok, Thailand: Chulalongkorn University Press, 2007, Appendix F pp 1-2.

[28] DEHN+SÖHNE, *Blitzplaner 3. aktualisierte Auflage*. Neumarkt, Germany: DEHN + SÖHNE GmbH + Co.KG., 2013, Avaliable: http://www.dehn.at/de-at/blitzplaner, 17 Mb and printed book 488 pages, pp. 71-72.

[29] P. Hasse J. Wiesinger and W. Zischank, *Handbuch fur Blitzschutz und Erdung*, 5 Auflage. Munchen, Germany: Pflaum Verlag 2006.

[30] Y Kishimoto "Consideration of protection angle method and lightning protection rules for side of high rise buildings in IEC 62305 standard" *The Institute of Electronics, Information and Communication Engineering* (*IEICE*) *Proceeding EMC 09,* Kyoto Japan, 2009.

[31] T.Horvath, "The protection space proved to be an undefined term" *2012 International Conference on Lightning Protection* (*ICLP*). pp. 4-5, Vienna, Austria, 2012.

[32] S.Ait-Amar and G.Berger, "A modified version of the rolling sphere method" *IEEE Transactions on Dielectrics and Electrical Insulation.*, vol. 16, no. 3, pp. 718-725, June. 2009.

[33] F.A.M.Rizk, "Modeling of lightning exposure of buildings and massive structures" *IEEE Transactions on Power Delivery.,* vol. 24, no. 4, pp. 1987-1998, Oct. 2009.

[34] A.Haddad and D.F.Warne, *Advances in High Voltage Engineering*, *IET Power and Series 49*. London, UK: The Institution of Engineering and Technology, 2007, pp. 77-130.

[35] P.N. Mikropoulus, and T.E. Tsovilis, "Estimation of lightning incidence to overheard transmission lines" *IEEE Transactions on Power Delivery.,* vol. 25, no. 3, pp. 1855-11865, July.2010.
[36] A.Borghetti, C.A.Nucci and M.Paolone, "Estimation of the statistical distribution of lightning current parameter at ground level from the data record by instrument tower" *IEEE Transactions on Power Delivery.,* vol. 19, no. 3, pp.1400-1409, July.2004.
[37] C.A.Nucci "A survey on cigre and ieee procedures for the estimation of the lightning performance of overhead transmission and distribution line" *X International Symposium on Lightning Protection* (*X-SIPDA*)., Curitiba, Brazil, 2009.
[38] K.Gieck, *Technische Formelsammlung.* Germany: Heilbronn, 1981.
[39] R.E. Larson, R.P. Hostetler and B.H. Edward, *Calculalus and Analytic Geometry 5th*. Canada: D.C. Health, 1994.
[40] J.Repka, *Calculalus and Analytic Geometry*. USA: Wm.C.Brown, 1994.
[41] A.Sultan, "An ordinary but surprisingly powerful theorem", *Australian Senior Mathematics Journal vol. 23*, no. 2, pp. 47-58, 2009.
[42] S.C. Chapra and R.P. Canale, *Numerical Methods for Engineers*. Singapore: McGraw-Hill (Asia), 2006, pp. 11-926.
[43] J. H. Wilkinson, *Rounding Errors in Algebraic Processes*. Dover Publications, 1994.
[44] Rajaraman V, *Computer oriented numerical methods*. India: Prentice-Hall of India Private Limited. New Delhi.
[45] Major General Luang Wichitwathakan, *History of Wat Khema*-pirataram, *Nonthaburi Province* (*fine art department of Ministry Culture, Thailand* 1971)., Bangkok, Thailand 1971, pp. 1-78.

Arnon singhasathein received B.Eng. and M.Eng. Degree in Electrical Engineering from King Mongkut's University of technology North Bangkok (KMUTNB), Thailand., He currently toward studying a Ph.D. Degree at King Mongkut' University of Technology North Bangkok (KMUTNB). His current research interested the lightning protection and high voltage engineering.

Weerapun Rungseevijitprapa received Dr. Ing. in Electrical Engineering from University of Hannover, Germany. He has been involved as an Associate Professor at Chulalongkorn University (CU), Bangkok Thailand. He is currently committee of Thai standard: (the protection against lightning) of the engineering institute of Thailand under H.M. the king's patronage. His current research interested the lightning protection, EMC and high voltage engineering.

Aphibal Pruksanubal received Dr. Ing. in Electrical Engineering from Technische University of Dresden, Germany. He has been elected an Associate Professor at King Mongkut's University of technology North Bangkok (KMUTNB), Thailand. His current research interested the lightning protection.