Torque Sensorless Decentralized Position/Force Control for Constrained Reconfigurable Manipulator via Non-fragile $H_{\infty}$ Dynamic Output Feedback

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Abstract – This paper studies the decentralized position/force control problem for constrained reconfigurable manipulator without torque sensing. A novel joint torque estimation scheme that exploits the existing structural elasticity of the manipulator joint with harmonic drive model is applied for each joint module. Based on the estimated joint torque and dynamic output feedback technique, a decentralized position/force control strategy is presented. In order to solve the problem of controller parameter perturbation, the non-fragile robust technique is introduced into the dynamic output feedback controller. Subsequently, the stability of the closed-loop system is proved using the Lyapunov theory and linear matrix inequality (LMI) technique. Finally, two 2-DOF constrained reconfigurable manipulators with different configurations are applied to verify the effectiveness of the proposed control scheme in numerical simulation.

Keywords: Constrained reconfigurable manipulator, Force/position control, Joint torque estimation, Non-fragile robust control, Dynamic output feedback

1. Introduction

Free motion control and constrained motion control are two basic strategies in the control of reconfigurable manipulators. Most of the existing researches for reconfigurable manipulators are carried out under the space which was totally free of environment constraints [1-4]. Actually, in most practical applications such as polishing, grinding, crawling, assembling etc., the manipulator may inevitably contact environments or manipulating objects [5-6]. Hence, position/force control is an important research topic in the domain of reconfigurable manipulator, particularly for applications in human environments and space.

In recent literatures, some researchers have paid much attention on controlling constrained manipulators [7-10]. Hybrid position/force control and impedance control are two basic strategies in the position/force control of constrained manipulators. The objective of hybrid position/force controller is to track a position (and orientation) trajectory in the position subspace and a force (and moment) trajectory in the force subspace. The impedance control of robot manipulators is to adjust the end-effector position in response to satisfy the target impedance behavior. In [11], a hybrid position/force trajectories control method for a robot manipulator interacting with a stiff environment is offered. A robust adaptive hybrid force/position control method for robot manipulators is presented in [12]. A force and position control method of parallel kinematic machines is discussed in [13], meanwhile, the artesian space computed torque control is applied to achieve force and position servoing directly in the task space. In [14], an impedance control strategy is presented to regulate both position and contact force of a piezoelectric–bimorph microgripper for micromanipulation and microassembly applications. In [15], continuum manipulator is designed to operate in constrained environments that are often unknown or unsensed, relying on body compliance to conform to obstacles. However, the aforementioned methods have been concentrated on the centralized control. For practical purposes, a centralized controller designed on the basis of an entire system may not be applicable for reconfigurable manipulators system due to its high computation costs, robustness, and complexities. Compared with centralized control, decentralized control scheme is more suitable for reconfigurable manipulators as it can effectively reduce the computational burden of centralized control structure. It should be stressed that, how to obtain each joint torque of constrained reconfigurable manipulator is a crucial issue for the purpose of achieving force decentralized control. Several techniques of direct joint torque sensing are proposed in the literatures [16-18]. However, using joint torque sensor measurements directly is known with many drawbacks and may degrade the reliability, ruggedness and simplicity of robot manipulators. Most robot manipulators are not equipped with joint torque sensor, and it is difficult
harmonic drive mechanism and the torque estimation algorithm are described in Section 2. The nonlinear interconnected subsystem dynamic model of constrained reconfigurable manipulator is described in Section 3. The non-fragile robust dynamic output feedback controller is designed and the stability proof is illustrated in detail based on Lyapunov theory in Section 4. The effectiveness of the presented method is verified by the numerical simulation results of two 2-DOF reconfigurable manipulators with different configurations in Section 5. Some conclusions are drawn in Section 6.

2. Torque Estimation Mechanism

Considering the constrained reconfigurable module manipulator consists of \( n \) -modules, each module provides an independently rotating joint with harmonic drive transmission. The graph in Fig. 1 depicts the main components of harmonic drives. The wave generator (WG) is connected to a motor, the circular spline (CS) is connected to the joint base, and the flexspline (FS) is sandwiched in between the CS and the WG and connected to the joint output [26].

Therefore, FS and WG torsion are defined as follows:

\[
\Delta \theta^f_i = \theta^f_{oi} - \theta^f_{si}
\]

![Fig. 1. Exploded view of a harmonic drive showing the three components](image1)

![Fig. 2. Kinematic representation of a harmonic drive showing the three ports](image2)
where $\theta_{fi}$ and $\theta_{go}$ denote the angular position at the flexspline gear side (gear-toothed circumference) and the flexspline angular position at the load side which is measured using the link-side encoder, respectively. $\theta_{gsi}$ and $\theta_{goi}$ denote the positions of the wave generator outside part (ball bearing outer rim) and the center part (wave generator plug), respectively.

When considering the compliance of wave generator, the angular positions at the components of the harmonic drive, as explained in [27], can be obtained by using the input/output kinematic relationship as:

$$\Delta \theta_i = \theta_{gi} - \theta_{goi} = -\delta \theta_{fi} \tag{3}$$

When the harmonic drive torsion is assumed to be caused by flexspline only, the harmonic drive torsion can be obtained by using the following expression:

$$\Delta \theta_i = \theta_{fi} - \theta_{goi} = -\frac{\theta_{gsi}}{\delta_i} \tag{4}$$

By adding and subtracting the terms $\theta_{fi}$ and $\theta_{goi}$ to (4), one obtains:

$$\Delta \theta_i = \Delta \theta_j - \Delta \theta_i \mid \delta_i + \theta_{ermi} \tag{5}$$

Where $\theta_{ermi}$ denotes the kinematic error of harmonic drive transmission. Based on the assumption that there is no relative motion between the wave generator output and the flexspline input, we have $\theta_{ermi} = 0$.

In order to model the harmonic drive compliance, consider its mechanical system analogy two-spring system [26]. As indicated in Fig. 3, the local elastic coefficient increases with the increase of $\tau_f$. Let us define the local elastic coefficient $L_{\beta}$ as:

$$L_{\beta} = \frac{d\tau_{\beta}}{d\Delta \theta_{\beta}} \tag{6}$$

Considering the symmetry property of the flexspline stiffness and using Taylor expansion, the local elastic coefficient is approximated by:

$$L_{\beta} = L_{\beta0} \left[ 1 + (a_f \tau_{\beta})^2 \right] \tag{7}$$

Where $L_{\beta0}$ and $a_f$ are constants to be determined. If $L_{\beta0} \neq 0$, then the flexspline torsion can be calculated as:

$$\Delta \theta_{\beta} = \int_0^{r_f} \frac{d\tau_{\beta}}{L_{\beta}} \tag{8}$$

Substituting $L_{\beta}$ of (7) into (8), we obtain:

$$\Delta \theta_{\beta} = \frac{\arctan(a_f \tau_{\beta})}{a_f L_{\beta0}} \tag{9}$$

where $\arctan(*)$ is the arctangent function. As shown in Fig. 3, the harmonic drive torsional deformation can range from $-\Psi/2$ to $-\Psi/2$ at zero torque output, but drops down to zero at rated torque, where $\Psi$ is the hysteresis loss. In order to replicate the hysteresis shape of this stiffness curve, the local elastic coefficient of wave generator is modeled as:

$$L_{gk} = L_{go} e^{a_g|\tau|} \tag{10}$$

Where $L_{go}$ and $a_g$ are constants to be determined. If $L_{go} \neq 0$, then the wave generator torsional angle can be calculated using the following relation:

$$\Delta \theta_g = \int_0^{r_g} \frac{d\tau_{gi}}{L_{gk}} \tag{11}$$

Substituting $L_{gk}$ of (10) into (11), we obtain:

$$\Delta \theta_g = \int_0^{r_g} \frac{\text{sign}(\tau_{gi})}{a_g L_{go}} \left[ 1 - e^{-a_g|\tau_{gi}|} \right] \tag{12}$$

Finally, by substituting the FS and WG deformation given in (9) and (12) into (5), the total deformation of the harmonic drive can be expressed as:

$$\Delta \theta_i = \frac{\arctan(a_f \tau_{gi})}{a_f \delta_i L_{go}} \left[ 1 - e^{-a_g|\tau_{gi}|} \right] \tag{13}$$

Then,
\[
\tau_q = \frac{1}{a} \tan \left( a \theta + \frac{\text{sign}(\tau_q)}{a} \right) \left( 1 - e^{-|\tau_q|} \right)
\]

(14)

where the wave generator torque \( \tau_q \) can be approximated by the motor torque command.

By the following formula, one can get the constrained torque which is obtained by the constrained force on the end-effector of manipulator:

\[
\tau_{ci} = \tau_{fio} - \tau_{fio}
\]

(15)

where \( \tau_{fio} \) denotes the joint torque which is obtained in free space, \( \tau_{fio} \) denotes the total joint torque in the constrained space. The total joint torque \( \tau_{fio} \) and/or joint torque in free space \( \tau_{fio} \) are directly obtained from formula (14) under the condition of constrained space and free space, respectively.

### 3. Problem Description

For a constrained reconfigurable manipulator, the motion of end-effector is constrained by its task environment. The constraint equation can be described as:

\[
\phi(q) = 0
\]

(16)

Under the environment constraint, the dynamic model of constrained reconfigurable manipulator with n-DOF can be described as follows:

\[
M(q)q + C(q,q)q + G(q) + F(q,q) = u + \tau_c
\]

(17)

where \( q \in R^n \) is the vector of joint displacements, \( M(q) \in R^{nxn} \) is the symmetric and positive definite inertia matrix, \( C(q,q) \in R^n \) is the matrix of centripetal Coriolis matrix, \( G(q) \in R^n \) is the gravity vector, \( u \in R^n \) is the control torque of output side in harmonic drive transmission, \( F(q,q) \in R^n \) denotes the joint friction force item, \( \tau_c \in R^n \) is the constrained torque which obtained by workspace force.

In the multidegree-of-freedom manipulator, the constrained torque vector for all joint \( \tau_c \) relates to the constrained force on the end-effector of manipulator \( f \) as follows:

\[
\tau_c = J_b^T(q) f
\]

(18)

where \( J_b^T(q) \in R^{nxn} \) is the Jacobian matrix. \( f \) denotes the constrained force on the end-effector of manipulator.

As the complexity of the reconfigurable manipulator system grew with the increase of the number of dof of the system model, how to realize decentralized force/position control is substantial important for reconfigurable manipulator [28-30]. Considering each of the joint module as a subsystem, the dynamics of module \( i \) can be expressed as:

\[
M_i(q_i)q_i + C_i(q_i,q_i)q_i + G_i(q_i)
\]

\[
+ F_i(q_i,q_i) + Z_i(q,q,q_i) = u_i + \tau_c
\]

(19)

\[
Z_i(q,q,q_i) = \sum_{j=1}^{n} M_j(q_j)q_j + [M_n(q) - M_i(q)] q_i
\]

\[
+ \left\{ \sum_{j=1}^{n} C_j(q_j,q_j)q_j + C_i(q_i,q_i)q_i - C_i(q_i,q_i)q_i \right\}
\]

\[
+ [\bar{G}_i(q) - G_i(q)]
\]

where \( q_i , \ q_i , \ \bar{G}_i(q) , \ F_i(q_i,q_i) , \ u_i \) and \( \tau_c \) are the \( i \) th element of the vectors \( q , q , \ G(q) , \ F(q,q) \), \( u \) and \( \tau_c \), respectively. \( M_i(q) \) and \( C_i(q,q) \) are the \( ij \) th element of the matrices \( M(q) \) and \( C(q,q) \), respectively.

The constrained reconfigurable manipulator dynamic model of nonlinear interconnected subsystem \( S_i \) can be presented by the following state equation:

\[
\begin{bmatrix}
    x_i \\
    y_i
\end{bmatrix} = \begin{bmatrix}
    A_i \\
    C_i
\end{bmatrix} \begin{bmatrix}
    x_i \\
    y_i
\end{bmatrix} + \begin{bmatrix}
    B_i \\
    y_i
\end{bmatrix} \begin{bmatrix}
    u_i \\
    f_i
\end{bmatrix} + \begin{bmatrix}
    g_i \\
    h_i
\end{bmatrix} \begin{bmatrix}
    \tau_c \\
    q_i
\end{bmatrix}
\]

(20)

where \( x_i = [x_{i,1} , x_{i,2} ]^T \) is the state vector of subsystem \( S_i \), and \( y_i \) is the output of subsystem \( S_i \). The matrices:

\[
A_i = \begin{bmatrix}
    0 & 1 \\
    0 & 0
\end{bmatrix}, \quad B_i = \begin{bmatrix}
    0 \\
    1
\end{bmatrix}, \quad C_i = \begin{bmatrix}
    1 & 0 \\
    0 & 1
\end{bmatrix},
\]

\[
f_i(q_i,q) = M_i^{-1}(q) [-C_i(q_i,q_i)q_i - G_i(q_i) - F_i(q_i,q_i)],
\]

\[
g_i(q_i,q) = M_i^{-1}(q),
\]

\[
h_i(q_i,q_i) = -M_i^{-1}(q_i) Z_i(q,q,q_i)
\]

The control objective is to design a decentralized position/force control scheme for Eq. (20) to make sure that the actual position \( \theta \) and the contact force \( f \) of the reconfigurable manipulator can follow their desired trajectories.

### 4. Control Design and Stability Analysis

In this section, a non-fragile robust dynamic output feedback controller with additive gain variations is designed to insure the asymptotic stability of the reconfigurable manipulator system as well as satisfying a predefined \( H_{\infty} \) performance level. Based on the Lyapunov theory, the
proving process of the proposed control method is presented.

An augmented system consisting of the system (20) and the integral of the tracking error \( e_\alpha = \int (y_{id} - y_i) \) is defined as follows:

\[
\begin{align*}
\begin{bmatrix}
\dot{x}_i \\
\tilde{x}_i \\
y_i 
\end{bmatrix}
&= \begin{bmatrix} 0 & -C_i \\
0 & A_i \\
I & 0 
\end{bmatrix} \begin{bmatrix}
x_i \\
\tilde{x}_i \\
y_i 
\end{bmatrix} + \begin{bmatrix} 0 \\
0 \\
0 
\end{bmatrix} \xi + R_i \tilde{y}_{id} \\
\end{align*}
\]

(21)

where

\[
\begin{align*}
A_i &= \begin{bmatrix} -C_i \\
0 \\
I 
\end{bmatrix}, \\
B_i &= \begin{bmatrix} 0 \\
0 \\
0 
\end{bmatrix}, \\
R_i &= \begin{bmatrix} I \\
0 \\
0 
\end{bmatrix}, \\
C_i &= \begin{bmatrix} 0 \\
0 \\
C_i 
\end{bmatrix}.
\end{align*}
\]

**Assumption 1:** The desired trajectories \( q_{id}, q_{ad} \) and \( q_{id} \) are bounded.

**Assumption 2:** The rigid environmental constraints are completely known. Moreover, the end-effector and constraint surface contact all the time, as well as the friction between them is ignored.

**Assumption 3:** The reconfigurable manipulator stays away from singularities to ensure Jacobian matrix full rank. Next, the RBF neural network with appropriate dimension is employed to approximate the nonlinear term \( f_i(q, q_i, W_i) \) and \( g_i(q, q_i) \) as follows [29]:

\[
\begin{align*}
f_i(q, q_i, W_i) &= W_i^T \Phi_i(q, q_i) + e_{i1} \| \tilde{x}_i \| \leq e_1 \\
g_i(q, q_i, W_i) &= W_i^T \Phi_i(q, q_i) + e_{i2} \| \tilde{x}_i \| \leq e_2
\end{align*}
\]

(22, 23)

where \( W_i \) and \( W_i \) are the ideal neural network weights, \( \Phi(*) \) is the neural network basis function, \( e_{i1} \) and \( e_{i2} \) are the neural network approximation errors, \( e_1, e_2 \) are known constants.

Defining \( \hat{W}_i \) as the estimations of \( W_i \) and \( \tilde{W}_i \) as estimation value of \( \hat{g}_i(q, q_i, W_i) \) and \( \hat{g}_i(q, W_i) \) is estimation value of \( g_i(q, q_i, W_i) \). \( \hat{f}_i(q, q_i, \tilde{W}_i) \) and \( \hat{g}_i(q, \tilde{W}_i) \) can be expressed as:

\[
\begin{align*}
\hat{f}_i(q, q_i, \tilde{W}_i) &= \tilde{W}_i^T \Phi_i(q, q_i) \\
\hat{g}_i(q, \tilde{W}_i) &= \tilde{W}_i^T \Phi_i(q, q_i)
\end{align*}
\]

(24, 25)

Define the estimation errors as \( \hat{W}_i = W_i - \tilde{W}_i \) and \( \tilde{W}_i = W_i - \hat{W}_i \). Therefore,

\[
\begin{align*}
f_i(q, q_i, W_i) - \hat{f}_i(q, q_i, \tilde{W}_i) &= \tilde{W}_i^T \Phi_i(q, q_i) + e_{i1} \\
g_i(q, W_i) - \hat{g}_i(q, \tilde{W}_i) &= \tilde{W}_i^T \Phi_i(q, q_i) + e_{i2}
\end{align*}
\]

(26, 27)

**Property [31]:** Boundedness of the interconnection term \( h_i(q, q_i, q_\alpha) \) for reconfigurable manipulator system:

\[
\| h_i(q, q_i, q_\alpha) \| \leq \sum_{j=1}^{n} d_j \| \tilde{q}_j \| \leq e_i
\]

(28)

where \( Q_i = 1 + \| q_i \| + \| \tilde{q}_j \| + \| q_\alpha \| , d_j \geq 0 \), \( \rho \geq 1 \).

Similarly, using the RBF neural networks to approximate the interconnection term as follows:

\[
\hat{h}_i(q_i, W_i) = W_i^T \Phi_i(q_i) + e_{i1} \| \xi \| \leq e_i
\]

(29)

where the \( W_i \) is the ideal neural network weights, \( \Phi_i(q) \) is the neural network basis function, \( e_{i1} \) is the neural network approximation errors, \( e_i \) is known constant.

Define \( \hat{W}_i \) and \( \tilde{W}_i \) as the estimations of \( W_i \) and \( \Phi_i(q) \). \( \hat{h}_i(q_i, \hat{W}_i) \) is the estimation value of \( h_i(q_i, \tilde{W}_i) \) and it can be expressed as:

\[
\hat{h}_i(q_i, \hat{W}_i) = \hat{W}_i^T \Phi_i(q_i)
\]

(30)

Define the estimation errors as \( \hat{W}_i = W_i - \tilde{W}_i \). Then, one obtained that:

\[
\hat{h}_i(q_i, \hat{W}_i) - \hat{h}_i(q_i, \tilde{W}_i) = \hat{W}_i^T \Phi_i(q_i) + e_{i1}
\]

(31)

Defining the approximation error as:

\[
\omega_{i1} = e_{i1} + e_{i2} (u_i + \tau_c)
\]

(32)

\[
\omega_{i2} = e_{i2}
\]

(33)

\[
\omega_i = [\omega_{i1}] + [\omega_{i2}]
\]

(34)

In order to approximate the performance index function, the weight vector should be updated as:

\[
\hat{W}_i = \eta_{i1} (Y \tilde{B}_i \tilde{x}_i - N \tilde{B}_i \tilde{x}_i) \Phi_i(q_i, q_\alpha)
\]

(35)

\[
\hat{W}_i = \eta_{i1} (Y \tilde{B}_i \tilde{x}_i - N \tilde{B}_i \tilde{x}_i) \Phi_i(q_i, u_i + \tau_c)
\]

(36)

\[
\hat{W}_i = \eta_{i1} (Y \tilde{B}_i \tilde{x}_i - N \tilde{B}_i \tilde{x}_i) \Phi_i(x_i)
\]

(37)

\[
\omega_i = \lambda_i (Y \tilde{B}_i \tilde{x}_i - N \tilde{B}_i \tilde{x}_i)
\]

(38)

where \( \eta_{i1}, \eta_{i2}, \eta_{i3} \) and \( \lambda_i \) are positive constants.

**Assumption 4:** The error between joint constrained torque estimated value and its actual value is small enough so that it can be ignored.

Considering the augmented constrained subsystems dynamic model (21), the decentralized non-fragile robust dynamic output feedback controller is designed as follows:
Lemma 2 ([33]). For any appropriate dimensions constant matrices $D$ and any scalar $\varepsilon > 0$, the following inequality holds:

$$D + \varepsilon I > 0.$$  

Combining Eq. (21) with Eq. (39), one can obtain:

$$A_0 = A_0 + \varepsilon I > 0.$$  

Next, define the joint error as follows:

$$\Delta = \begin{bmatrix} x \end{bmatrix} - \hat{x} = \begin{bmatrix} x \end{bmatrix} - \Delta.$$  

where $x = T + \hat{x}$ and $\hat{x}$ are the nominal controller gain matrices.

Theorem. Based on Lemma 2, given $\gamma > 0$ and matrices $M$ and $N$ satisfy $M^T \gamma M = - X$, the following conditions are known real constant matrices of appropriate dimensions $D, E$ and any scalar $\varepsilon > 0$.

$$A_0 = A_0 + \varepsilon I > 0.$$  

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where $x = T + \hat{x}$ and $\hat{x}$ are the nominal controller gain matrices.
and satisfies the $H_\infty$ performance indicator as follows:

$$\|e\| \leq \gamma^2 \|e\| + V(0) \quad (57)$$

where $\|e\| = \int_0^t (e^T e) dt$, $\|u\| = \int_0^t (d^T d) dt$.

**Proof:** Choosing the Lyapunov candidate as $V_i = x_i^T P_i x_i$, the time derivative of $V_i$ along the trajectories of system (43) is:

$$V_i = x_i^T \left( A_i^T P_i + P_i A_i \right) x_i + x_i^T P_i B_i d_i$$

$$+ d_i^T B_i^T P_i x_i + \left( \gamma \left( y - \bar{y} \right) - \nu \hat{y} \right) d_i$$

$$+ \eta_i \left( \hat{y} \hat{y} - \eta \right) + \eta_i \left( \hat{y} \hat{y} - \eta \hat{y} \right)$$

$$- \lambda_i \omega_i + \omega_i \eta_i - \lambda_i \omega_i + \omega_i \eta_i$$

Set $P_i = \begin{bmatrix} Y & N \hline N^T & V \end{bmatrix}$, then:

$$\dot{V}_i = x_i^T \left( A_i^T P_i + P_i A_i \right) x_i + x_i^T P_i B_i d_i$$

$$+ d_i^T B_i^T P_i x_i + \left( \gamma \left( y - \bar{y} \right) - \nu \hat{y} \right) d_i$$

$$+ \eta_i \left( \hat{y} \hat{y} - \eta \right) + \eta_i \left( \hat{y} \hat{y} - \eta \hat{y} \right)$$

$$- \lambda_i \omega_i + \omega_i \eta_i - \lambda_i \omega_i + \omega_i \eta_i$$

Substituting (35)-(37) into (59), yields the expression:

$$V_i = x_i^T \left( A_i^T P_i + P_i A_i \right) x_i + x_i^T P_i B_i d_i$$

$$+ d_i^T B_i^T P_i x_i + \left( \gamma \left( y - \bar{y} \right) - \nu \hat{y} \right) d_i$$

$$+ \eta_i \left( \hat{y} \hat{y} - \eta \right) + \eta_i \left( \hat{y} \hat{y} - \eta \hat{y} \right)$$

$$- \lambda_i \omega_i + \omega_i \eta_i - \lambda_i \omega_i + \omega_i \eta_i$$

By utilizing the adaptive laws (38), one has:

$$\dot{V}_i = x_i^T \left( A_i^T P_i + P_i A_i \right) x_i + x_i^T P_i B_i d_i$$

$$+ d_i^T B_i^T P_i x_i + \left( \gamma \left( y - \bar{y} \right) - \nu \hat{y} \right) d_i$$

$$+ \eta_i \left( \hat{y} \hat{y} - \eta \right) + \eta_i \left( \hat{y} \hat{y} - \eta \hat{y} \right)$$

$$- \lambda_i \omega_i + \omega_i \eta_i - \lambda_i \omega_i + \omega_i \eta_i$$

$$+ \eta_i \left( \hat{y} \hat{y} - \eta \right) + \eta_i \left( \hat{y} \hat{y} - \eta \hat{y} \right)$$

$$- \lambda_i \omega_i + \omega_i \eta_i - \lambda_i \omega_i + \omega_i \eta_i$$

Given the following index:

$$J = \int_0^t (e^T e - \gamma^2 d^T d) dt$$

Thus,

$$J = \int_0^t (e^T e - \gamma^2 d^T d) dt - \int_0^t \dot{V}_i dt$$

$$\leq \int_0^t (e^T e - \gamma^2 d^T d) dt + V(0)$$

where

$$e^T e = (x_i - \bar{x}_i, y - \bar{y})^T (x_i - \bar{x}_i, y - \bar{y})$$

Let $E = \begin{bmatrix} 0 & I \end{bmatrix}$, one can obtain:

$$e^T e = d_i^T E^T d_i - d_i^T E^T C_d x_i - x_i^T C_d^T E d_i + x_i^T C_d^T E d_i$$

Based on Eq.(62), for achieve the required property index (59) and stability of the augmented system (39) the following inequality needs to be satisfied:

$$V_i + e^T e - \gamma^2 d^T d < 0 \quad (64)$$

By using Eq. (58), Eq. (63) and based on Lemma 2, the following formula must establish:

$$\begin{bmatrix} A_i^T P_i + P_i A_i & P B_i - C_i E & C_i^T \\ * & E^T E - \gamma^2 I & 0 \end{bmatrix} < 0 \quad (65)$$

Inequality (65) can further decomposed as below:

$$\begin{bmatrix} A_i^T P_i + P_i A_i & P B_i - C_i E & C_i^T \\ * & E^T E - \gamma^2 I & 0 \end{bmatrix} < 0 \quad (66)$$

On the basis of Lemma 1, inequality (66) is implied by the following formula:

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\[
\begin{bmatrix}
A'\lambda P_j + P_j A'\mu & P_j B'w & C'w - C'w 0 \\
* & E'E - \gamma^2 I & 0 \\
* & * & -I 0 \\
* & * & * - \epsilon^{-1} 0 \\
* & * & * - \epsilon^{-1} - \epsilon^{-1}
\end{bmatrix} < 0 \quad (67)
\]

Set [35]:
\[
F_1 = \begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix}, \quad F_2 = \begin{bmatrix} I & Y \\ 0 & N^T \end{bmatrix}
\]

then, it is clear to see that \( F_1^T P_1 = F_2^T \). Pre- and post-multiplying inequality (67) by \( F_1^T F_1 \) and its transpose respectively, yields:
\[
\begin{bmatrix}
F_1^T A'F_1 + F_1^T A'F_1 & F_1^T B'w & F_1^T C'w - F_1^T C'w 0 \\
* & E'E - \gamma^2 I & 0 \\
* & * & -I 0 \\
* & * & * - \epsilon^{-1} 0 \\
* & * & * - \epsilon^{-1} - \epsilon^{-1}
\end{bmatrix} < 0 \quad (68)
\]

According to Eq. (53)-(56), it follows that (68) implies (49). Therefore, the system satisfies the \( H_\infty \) performance indicator and this completes the proof of Theorem.

5. Numerical simulations

To verify the effectiveness of the proposed non-fragile robust dynamic output feedback control strategy, in this subsection, two 2-DOF constrained reconfigurable manipulators with different configurations (see Fig.4) are employed.

Suppose the additive gain variations of the controller are expressed by Eq. (40) with:
\[
L_{ai} = L_{bi} = \begin{bmatrix}
0.3 \\
0.5 \\
-0.1 \\
0.2
\end{bmatrix}, \quad M_{ai} = M_{bi} = \begin{bmatrix}
-0.2 & 0.4 & 0.1 & 0.3
\end{bmatrix}
\]

\[
L_{ci} = L_{di} = 0.1, \quad M_{ci} = M_{di} = \begin{bmatrix}
0.2 & 0.6 & 0.3 & 0.5
\end{bmatrix}
\]

\[
F_a = I, \quad \gamma = 0.002, \quad \eta_\lambda = 0.002, \quad \gamma = 1.0.
\]

By using YALMIP Toolbox [36] and selecting LMILab solver under the Matlab software, one can obtain the controller gain matrices listed below:
\[
A_w = A_{2w} = \begin{bmatrix}
-10.3049 & -18.5541 & -1.0296 & -9.5123 \\
-0.4121 & -1.3049 & 0 & 0 \\
-0.3600 & 0 & -9.7638 & -1.4081 \\
-0.4759 & 0 & -1.7485 & -9.4442
\end{bmatrix}
\]
\[
B_{w} = B_{2w} = \begin{bmatrix}
-2.4495 & 3.4799 & 1.7963 & 3.8811 \\
3.8811 & -2.4495 & -1.7963 & 3.8819 \\
1.0297 & -1.0297 & -1.5868 & -0.5360 \\
3.1680 & 0.1650 & -4.9985 & -8.7807
\end{bmatrix}
\]
\[
C_{w} = C_{2w} = \begin{bmatrix}
-0.6980 & 0.6980 & -0.5111 & -0.1360
\end{bmatrix}
\]
\[
D_{w} = D_{2w} = \begin{bmatrix}
2.0889 & 2.0889 & -1.6513 & -365.30
\end{bmatrix}
\]

For the sake of analyzing aforementioned configurations, a form of analytic diagram is offered in Fig. 5.

The control law (39) is applied to the whole control system and the initial position and velocity are set as \( q_1(0) = q_2(0) = 0 \) and \( q_1(0) = q_2(0) = 0 \), respectively. The control parameters are selected as \( \eta_\gamma = 0.002, \eta_\lambda = 0.002, \gamma = 500, \lambda = 0.002 \) and the \( H_\infty \) performance indicator is defined as \( \gamma = 1.0 \).

Fig.4 (a) shows the planar graph of configuration a. It is assumed that the constrained reconfigurable manipulator is simulating a polishing task and the constraint equation can be depicted as:
\[
\phi(q) = l_1 \cos(q_1) + l_2 \cos(q_2) - 1 = 0 \quad (69)
\]

where \( l_1 = 1, \ l_2 = 1 \) are the lengths of the two robot links. The desired position and the desired constraint force are defined as:
\[
q_{1d} = \cos(\pi t) + \sin(\pi t) \\
q_{2d} = \arccos\left(\frac{1 - l_1 \cos(q_{1d})}{l_2}\right)
\]
Position tracking trajectories and position tracking errors of configuration \( a \) are illustrated in Fig. 6 where the actual and desired trajectories are almost overlap. From Fig. 7, one can obtain that mapping the end contact force to joint space, the equivalent torque tracking errors can also converge to zero. Fig. 8 shows the tracking performance of the contact force and the force tracking error curves, in which the system converges to an error that can be considered as zero (less than 0.01 N). It can be concluded that high performance of the simultaneous position-force tracking is achieved with relatively smooth control effort.

In order to further check the effectiveness of the presented scheme for decentralized position/force control under different configurations, the same controller is used to configuration \( b \). Fig. 4 (b) shows that the reconfigurable manipulator works in constrained environment such as cylinders. The constraint equation can be expressed as \( \phi(q) = l_1 + l_2 \cos(q_2) - 1.5 = 0 \). The desired position and the desired constraint force are defined as:

\[
q_1 = \cos(2t)
\]

\[
q_2 = \frac{\pi}{3}
\]

\[
f_d = 6 \text{ N}
\]

The parameters of the decentralized position/force controller in configuration \( b \) are chosen as those in configuration \( a \). The position tracking performance, equivalent torque tracking performance and force tracking performance of configuration \( b \) are presented in Figs. 9-11, respectively. The numerical simulation results demonstrate that the presented decentralized position/force control strategy can be used to different configurations of constrained reconfigurable manipulators without changing any control parameters. This is an important and meaningful advantage for control the reconfigurable manipulators with a requirement of frequent conversion from one configuration.
to another in order to complete different kinds of tasks in different environment.

**Fig. 8.** Force tracking curves and force tracking error of configuration a

**Fig. 9.** (a) Position tracking performance of configuration b; (b) Position tracking error of configuration b

**Fig. 10.** (a) Equivalent torque of configuration b; (b) Equivalent torque tracking error of configuration b

**Fig. 11.** Force tracking curves and force tracking error of configuration b
6. Conclusion

A torque sensorless decentralized position/force control scheme using dynamic output feedback technique was presented for constrained reconfigurable manipulator. Torque estimation based on position measurements provides an advantage of noise immunity to the estimated joint torque and reduces the cost of joint torque sensing. The influence of parameter perturbation of dynamic output feedback controller with additive gain variations has been reduced by introducing non-fragile robust method. Moreover, the stability of closed-loop system is proved using the Lyapunov theory and linear matrix inequality (LMI) technique. Finally, the effectiveness of the proposed scheme was verified under the conditions of different configurations without revising any parameters.

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References


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