Systematic Dynamic Modeling of an Integrated Single-stage Power Converter

Ki-Young Choi*, Kui-Jun Lee**, Yong-Wook Kim* and Rae-Young Kim†

Abstract – This paper proposes a novel systematic modeling approach for an integrated single-stage power converter in order to predict its dynamic characteristics. The basic strategy of the proposed modeling is substituting the internal converters with an equivalent current source, and then deriving the dynamic equations under a standalone operation using the state-space averaging technique. The proposed approach provides an intuitive modeling solution and simplified mathematical process with accurate dynamic prediction. The simulation and experimental results by using an integrated boost-flyback converter prototype provide verification consistent with theoretical expectations.

Keywords: Integrated single-stage power converter, Systematic modeling, State-space averaging, Small-signal model, Dynamic equation

1. Introduction

For low cost, high efficiency, and high performance, integrated power converter topology that incorporates a multi-stage power converter into an equivalent single-stage has been extensively researched [1-4]. One typical example is the integrated boost (or buck) flyback converter (IBFC) for dc-dc conversion, electronic ballast, and power-factor-corrected power supply application.

As shown in Fig. 1, the IBFC integrates two single converters: a boost (or buck) converter and a flyback converter under a cascade configuration, into a single-stage configuration with a shared main switch, \( S_p \), and a dc-link capacitor, \( C_p \). As a result of topological integration, the IBFC provides several advantages over a conventional multi-stage converter, including reduced size, weight, and cost, and better power conversion efficiency [2, 15].

Moreover, the control performances, such as the input-current shaping, isolation, and fast output regulation, can be improved by a proper single feedback compensator design that includes accurate prediction of the IBFC’s dynamic behaviors [5, 6]. However, the prediction of these behaviors, which is essential to guarantee the compensator’s dynamic performance and stability, is abstruse to the due increasing number of components and complicated structure. As a result, the compensator has often been designed without a theoretical analysis and rationale. Furthermore, when integrated boost, buck, or flyback converters inside the IBFC are operated under discontinuous (DCM) or continuous (CCM) conduction

![IBFC Diagram](http://dx.doi.org/10.5370/JEET.2015.10.5.1921)

**Fig. 1.** Typical examples of the integrated single-stage power converter.

modes, prediction of such dynamic behaviors becomes even more difficult.

Since establishing the average concept to remove the trivial switching effect of a power converter by Middlebrook in 1974, several modeling approaches, including the well-known state-space averaging method and the averaged switch model, have been proposed for the prediction of converter dynamics [7-13]. These approaches have usually showed effective prediction results for most existing power converters, and thus some papers have explored a direct extension into the IBFC [14, 15]. However, it exhibits some disadvantages, which include the complexity of the
circuit operation analysis and the extensive efforts required for mathematical calculation.

As an alternative, the converter-integration approach, which analyzes two internal converters under a standalone operation condition by using the average concept, has been introduced [1, 16], and [17]. These approaches greatly simplify the analysis work, but lack accuracy as they do not account for interactional behaviors. Furthermore, no concrete results supported by a theoretical rationale have been reported. Terminal network approaches, such as the graft scheme and the five-terminal switched transformer average model, have been introduced [18, 19]. These approaches treat the switching element as the port network and incorporate the network parameters to improve prediction accuracy. Compared to the converter-integration approach, however, they require additional consideration for the network parameters and extensive mathematical efforts to satisfy more accurate analysis by increasing the number of terminals.

In this paper, a novel systematic modeling approach that provides accurate dynamics prediction is proposed in order to achieve a theoretical single feedback compensator design for an integrated single-stage power converter. The proposed approach substitutes the internal converter with an equivalent current sinking or sourcing and then combines them to construct a complete dynamic equation using the state-space averaging concept. By using this methodology, the modeling approach becomes straightforward, and the mathematical effort is significantly reduced, while still providing accurate converter behaviors including the interactional dynamics of the internal converters. A detailed modeling procedure is presented, specifically on the target of an integrated boost-flyback converter, as a typical example of an integrated power converter, and a small-signal model of the full fourth-order system is derived. Based on this derivation, a single feedback compensator is designed with reasonable dynamic response and stability. Several simulation and experiment results, based on a 100 W integrated boost-flyback converter prototype, are provided in order to verify the accuracy of the proposed approach and its effectiveness.

2. Interactional Behaviors of the Integrated Power Converter

The topological incorporation of the integrated single-stage power converter inherently raises problems of complicated dynamics among the internal converters. For a simple explanation, taking the integrated boost-flyback converter (IBoFC) as a convenient example, the instantaneous current waveforms during one switching period, $T_s$, are illustrated in Fig. 2. The values $i_{lh}$ and $i_{lm}$ denote the boost and magnetizing inductor currents; $i_{boost}$ and $i_{flyback}$ denote the diode $D_2$ and the flyback transformer currents; and $i_{ce}$ is the dc-link capacitor current. Note that the waveforms are distinguished by three different submodes in one switching period. In mode 1, the switch $S_1$ turns on, and the $i_{lh}$ and the $i_{lw}$ are linearly increased. The $i_{boost}$ is zero since the diode $D_2$ is reverse-biased. The $i_{flyback}$ is the same as the $i_{lm}$. In mode 2, the switch $S_1$ turns off and the diode $D_2$ is on. The $i_{lh}$ decreases linearly and flows through diode $D_2$. Due to the switch $S_1$ being off, the $i_{flyback}$ becomes zero. Mode 3 starts when the $i_{lh}$ is zero with DCM. The value of $i_{lh}$ and $i_{boost}$ maintain zero, while $i_{lm}$ flows continuously with CCM.

As illustrated in Fig. 2, the IBoFC operates similar to the standalone boost and flyback converter. However, the $i_{ce}$, which determines the dc-link capacitor voltage, $v_{ce}$, is alternatively governed by the $i_{lh}$ and the $i_{lm}$. Consequently, the $v_{ce}$ varies according to the internal converter operations. The overall converter dynamics become complicated due to the interactional behaviors originating from the inside converters.

3. Novel systematical modeling approach

As seen from Fig. 2, the total electrical charge, $Q_{ce}$,
incoming to $C_e$ over one switching period is

$$ q_{ce} = \int_0^T i_{ce} \, dt = \int_0^T (-i_{f\text{flyback}}) \, dt + \int_0^T i_{\text{boost}} \, dt $$

(1)

Applying the small ripple approximation under the assumption that the switching ripple is smaller than the dc component, in our case the $i_{Lm}$ peak-to-peak ripple percentage is about 28% based on Eq. (2) and Table 1, where "—" designates the averaged value over one switching period, and taking the average operation over one switching period, the averaged dc-link capacitor voltage, $\bar{v}_{Ce}$, is given as Eq. (3).

$$ i_{Lm\_pp} (%) = \frac{i_{Lm\_pp}}{i_{Lm}} \times 100 = \frac{\bar{v}_{Ce} \cdot V \cdot d (1 - d) T_s}{L_m \cdot P_n} \times 100 $$

(2)

$$ C_e \frac{dv_{Ce}}{dt} = -\bar{i}_{f\text{flyback}} + \bar{i}_{\text{boost}} $$

(3)

Eq. (3) indicates that the dc-link capacitor is modeled by the averaged boost diode and the flyback currents, $\bar{i}_{\text{boost}}$ or $\bar{i}_{f\text{flyback}}$. Therefore, from the state-space averaging point of view, the internal converters are equivalently approximated as the corresponding current sinking or sourcing as shown in Fig. 3. The modeling equations of the internal converters structures are simply obtained using Kirchhoff’s circuit laws and then combined in order to construct the full order modeling equations for the integrated boost-flyback power converter. Such a modeling approach using a current source significantly simplifies the model effort, while providing a straightforward solution that provides accurate dynamics including interactive behaviors.

3.1 Detail modeling procedure

Fig. 4 shows the equivalent standalone model of the internal boost converter and its inductor current under DCM operation. Note that the internal flyback converter is represented by the current sinking.

According to each subinterval, the state equations can be obtained as

![Fig. 3. Equivalent standalone model of the internal converter with a current source.](image)

![Fig. 4. Operational mode of the internal boost converter.](image)
Subinterval 1
\[
\begin{align*}
L_b \frac{di_{bh}}{dt} &= v_{in} \\
C_e \frac{dv_{Ce}}{dt} &= -\tau_{flyback} \\
\end{align*}
\]
and
\[
\begin{align*}
L_b \frac{di_{bh}}{dt} &= v_{in} - v_{Ce} \\
C_e \frac{dv_{Ce}}{dt} &= i_{Lb} - \tau_{flyback}
\end{align*}
\]

Subinterval 2
\[
\begin{align*}
L_b \frac{di_{bh}}{dt} &= 0 \\
C_e \frac{dv_{Ce}}{dt} &= -\tau_{flyback}
\end{align*}
\]

From Eq. (4) and Fig. 4(d), the averaged state equations can be obtained as
\[
\begin{align*}
L_b \frac{di_{bh}}{dt} &= v_{in} \cdot d + (v_{in} - v_{Ce}) \cdot d_a + 0 \cdot d_b \\
C_e \frac{dv_{Ce}}{dt} &= -\tau_{flyback} \cdot d + \tau_{boost} \cdot d_a - \tau_{flyback} \cdot d_b
\end{align*}
\]

where \( \tau_{boost} \) is used instead of \( i_{Lb} \cdot d_b \) because the small ripple approximation cannot be applied to \( i_{Lb} \) due to DCM operation.

In Eq. (5), the subinterval time \( d_a \) and \( d_b \) and \( \tau_{boost} \) should be replaced by an expression of the state and input variables. In Fig. 4(d), the maximum value of \( i_{Lb} \) (\( i_{Lb \text{peak}} \)) and the average value of \( i_{Lb} \) (\( i_{Lb} \)) are given by
\[
\begin{align*}
i_{Lb \text{peak}} &= \frac{v_{in} \cdot T_s}{L_b} \\
i_{Lb} &= \frac{1}{2} (d + d_a) \cdot i_{Lb \text{peak}}
\end{align*}
\]

where \( T_s \) is the switching period. From Eqs. (6) and (7), the relational expression between \( d_a \) (or \( d_b \)) and \( d \) can be obtained as
\[
\begin{align*}
d_a &= \frac{2 \tau_{Lb} L_b f_s}{v_{in} d} - d = q - d \\
q &= \frac{2 \tau_{Lb} L_b f_s}{v_{in} d} \\
d_b &= 1 - (d + d_a) = 1 - q
\end{align*}
\]

where \( f_s \) is the switching frequency (=1/\( T_s \)). Furthermore, the average boost current \( \bar{i}_{boost} \) can be obtained by calculating the triangle area during subinterval 2 in Fig. 4(d).
\[
\bar{i}_{boost} = \frac{1}{2} (q - d) \cdot \frac{v_{in} \cdot T_s}{L_b}
\]

From Eqs. (7) and (9), the expression between \( i_{Lb} \) and \( \bar{i}_{boost} \) can be obtained as
\[
\bar{i}_{boost} = (q - d) \cdot \frac{i_{Lb} - \bar{i}_{flyback}}{q}
\]

By applying Eqs. (8) and (10) to Eq. (5), the averaged state equations of the boost converter consist only of the state and input variables as
\[
\begin{align*}
L_b \frac{di_{bh}}{dt} &= v_{in} \cdot q - v_{Ce} \cdot (q - d) \\
C_e \frac{dv_{Ce}}{dt} &= i_{Lb} \cdot \frac{(q - d)}{q} - \bar{i}_{flyback}
\end{align*}
\]

Fig. 5 shows the equivalent standalone model of the internal flyback converter and the current waveform. Similarly, the internal boost converter is represented by the current sourcing.

According to each subinterval, the state equations can be obtained as

Subinterval 1
\[
\begin{align*}
C_e \frac{dv_{Ce}}{dt} &= \bar{i}_{boost} - i_{Lm} \\
L_m \frac{di_{Lm}}{dt} &= v_{Ce} \\
C_o \frac{dv_{o}}{dt} &= \frac{v_{o}}{R}
\end{align*}
\]

Subinterval 2
\[
\begin{align*}
C_e \frac{dv_{Ce}}{dt} &= \bar{i}_{boost} \\
L_m \frac{di_{Lm}}{dt} &= -\frac{v_{o}}{n} \\
C_o \frac{dv_{o}}{dt} &= \frac{i_{Lm}}{n} - \frac{v_{o}}{R}
\end{align*}
\]

From Eq. (12) and Fig. 5(c), the averaged state equations of the flyback converter are given by
\[
\begin{align*}
C_e \frac{dv_{Ce}}{dt} &= (\bar{i}_{boost} - i_{Lm}) \cdot d + \tau_{boost} \cdot (1 - d) \\
L_m \frac{di_{Lm}}{dt} &= v_{Ce} \cdot d - \frac{v_{o}}{n} \cdot (1 - d) \\
C_o \frac{dv_{o}}{dt} &= \frac{v_{o}}{R} \cdot d + \left( \frac{i_{Lm} - v_{o}}{n} \right) \cdot (1 - d)
\end{align*}
\]

By comparing Eq. (11) and Eq. (13), it can be determined that the analytical expressions of the two equivalent current sources are
\[
\bar{i}_{boost} = \frac{i_{Lb} \cdot (q - d)}{q}, \quad \bar{i}_{flyback} = \frac{i_{Lm} \cdot d}{q}
\]

Therefore, the complete averaged state equations of the IBoFC are given by Eq. (15) from (11), (13), and (14).
small ac variation as follows:

\[
\begin{align*}
\dot{I}_{\text{Lm}} &= 0 - \frac{q - d}{L_b} I_{\text{Lb}} - \frac{d}{C_e} q - 0 - 0 \left[ \begin{array}{c} I_{\text{Lb}} \\ V_{\text{Ce}} \\ I_{\text{Lm}} \\ V_o \end{array} \right] \\
\frac{d}{dt} \left[ \begin{array}{c} I_{\text{Lb}} \\ V_{\text{Ce}} \\ I_{\text{Lm}} \\ V_o \end{array} \right] &= \left[ \begin{array}{cccc} 0 & -\frac{q - d}{L_\text{b}} & 0 & 0 \\ \frac{q - d}{C_\text{e}} q & 0 & -\frac{d}{C_\text{e}} & 0 \\ 0 & \frac{d}{L_\text{m}} & 0 & 1 - d - \frac{1 - d}{L_\text{m} \cdot n} - \frac{1 - d}{C_\text{o} \cdot n} - \frac{1 - d}{C_\text{o} \cdot R} \\ 0 & 0 & 1 - \frac{1 - d}{C_\text{o} \cdot n} - \frac{1 - d}{C_\text{o} \cdot R} & 0 \end{array} \right] \left[ \begin{array}{c} I_{\text{Lb}} \\ V_{\text{Ce}} \\ I_{\text{Lm}} \\ V_o \end{array} \right] + \left[ \begin{array}{c} q \\ 0 \\ 0 \\ 0 \end{array} \right] v_m \\
\end{align*}
\]

Subsequently, the perturbed expressions such as Eq. (16) are applied to Eq. (15), where \( X = (I_{\text{Lb}}, V_{\text{Ce}}, I_{\text{Lm}} \text{ and } V_o) \) is the dc quiescent value and \( \delta = (\dot{I}_{\text{Lb}}, \dot{V}_{\text{Ce}}, \dot{I}_{\text{Lm}} \text{ and } \dot{V}_o) \) is the small ac variation as follows:

\[
\begin{align*}
\begin{bmatrix} I_{\text{Lb}} \\ V_{\text{Ce}} \\ I_{\text{Lm}} \\ V_o \end{bmatrix} &= \begin{bmatrix} I_{\text{Lb}} \\ V_{\text{Ce}} \\ I_{\text{Lm}} \\ V_o \end{bmatrix} + \begin{bmatrix} I_{\text{Lb}} \\ V_{\text{Ce}} \\ I_{\text{Lm}} \\ V_o \end{bmatrix} \\
\dot{v}_m &= V_{\text{in}} + \dot{v}_m \\
\end{align*}
\]

If the second-order ac terms are neglected from the resultant equations, the dc terms and first-order ac terms remain. The resultant first-order ac terms that correspond to the small-signal ac model are given by Eq. (17), where \( \dot{v}_o \) is the ac variation component of the load current.

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} I_{\text{Lb}} \\ \dot{V}_{\text{Ce}} \\ \dot{I}_{\text{Lm}} \\ \dot{V}_o \end{bmatrix} &= \begin{bmatrix} \frac{k(V_{\text{in}} - V_{\text{Ce}})}{L_b} - \frac{D - kI_{\text{Lb}}}{L_b} & 0 & 0 & 0 \\
0 & 0 & -\frac{D}{C_e} & 0 \\
0 & 0 & 0 & -\frac{1 - D}{L_m \cdot n} \\
0 & 0 & 0 & 0 - \frac{1 - D}{C_o \cdot n} - \frac{1 - D}{C_o \cdot R} \\
\end{bmatrix} \begin{bmatrix} I_{\text{Lb}} \\ \dot{V}_{\text{Ce}} \\ \dot{I}_{\text{Lm}} \\ \dot{V}_o \end{bmatrix} + \begin{bmatrix} kI_{\text{Lb}} (V_{\text{Ce}} - V_{\text{in}} + \frac{D \cdot V_{\text{Ce}}}{kL_b}) & 0 & 0 & 0 \\
-\frac{1}{C_e} (I_{\text{Lm}} + \frac{2}{k} \dot{I}_{\text{Lm}}) & 0 & 0 & 0 \\
\frac{1}{L_m} (V_{\text{Ce}} + \frac{V_o}{n}) & 0 & 0 & 0 \\
-\frac{I_{\text{Lm}}}{C_o \cdot n} & 0 & -1 \end{bmatrix} \begin{bmatrix} I_{\text{Lb}} \\ \dot{V}_{\text{Ce}} \\ \dot{I}_{\text{Lm}} \\ \dot{V}_o \end{bmatrix} \\
\end{align*}
\]

Furthermore, based on the resultant dc terms and the parameter values in Table 1, the equilibrium dc values are obtained as \( I_{\text{Lb}} = 3.333A, V_{\text{Ce}} = 58.904V, I_{\text{Lm}} = 4.198A, \) and \( D = 0.404. \) These test conditions shown in Table 1 were chosen to implement a high step-up dc-dc converter which is required in recent distributed generation systems, with a low input voltage [20-24].

Fig. 6 shows the Bode plot of the control-to-output transfer function \( (G_{\text{cl}} = \dot{v}_o / \dot{d}) \) and the output impedance \( (Z_{\text{out}} = \dot{v}_o / \dot{d}) \).

<table>
<thead>
<tr>
<th>Table 1. Test conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Power (P_o)</td>
</tr>
<tr>
<td>Input Voltage (V_i)</td>
</tr>
<tr>
<td>Output Voltage (V_o)</td>
</tr>
<tr>
<td>Switching Frequency(f)</td>
</tr>
<tr>
<td>Boost Inductor (L_b)</td>
</tr>
<tr>
<td>Magnetizing inductor (L_m)</td>
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<tr>
<td>DC-link Capacitor (C_e)</td>
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<tr>
<td>Output Capacitor (C_o)</td>
</tr>
<tr>
<td>Transformer turns ratio (n)</td>
</tr>
</tbody>
</table>

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In the previous literature [14], the direct extension approach of the conventional state-space averaging method is applied to obtain the dynamic model of a single-stage single-switch (S4) parallel boost-flyback-flyback converter. The average state variable description of the converter is derived at first. However, some inductor currents of total five energy storage elements aren’t selected as state variables since those inductor currents operate in DCM. Furthermore, during final small-signal model derivation, one capacitor voltage is considered as a constant value and omitted from the vector of state variables to simplify the procedure since this variable is just an interactional dynamic. Therefore, the complete dynamic model isn’t achieved. Moreover, the validation of the derived model is verified only in the time domain, not the frequency domain.

3.2 Single loop compensator design

Fig. 7 shows a block diagram of the output voltage control based on the small-signal model, and Eq. (18) is the analytical expression for the output voltage. \( G_{vd}(=\hat{v}_o/\hat{v}_{in}) \) is the line-to-output transfer function, \( G_{pwm} \) is the pulse-width modulator gain, \( G_c \) is the voltage compensator, \( H \) is the sensor gain, and \( T \) is the loop gain.

\[
\hat{v}_o = \hat{v}_{in} \frac{T}{H} + \frac{G_{vd}}{1+T} \hat{v}_{in} G_{pwm} \frac{Z_{out}}{1+T} T = G_c G_{pwm} G_{vd} H
\]

For voltage regulation, a proportional-integral (PI) compensator is designed based on the control-to-output transfer function. A typical PI compensator can be adopted with the following design process:

1) place one pole to eliminate the steady-state error (integrator);
2) place one zero in the low frequency region (at 10 Hz) to secure the phase margin in front of the resonant frequency (2.24 kHz);
3) locate the crossover frequency approximately one decade less than the resonant frequency (at 100 Hz);
4) determine the dc gain. By using this process, the 100 Hz bandwidth and the 85° phase margin are secured, and the resultant PI compensator is

\[
G_c = k \frac{1+s/\omega_z}{s}, \quad (k = 4.0192, \omega_z = 2\pi \cdot 10)
\]

Fig. 7. Block diagram of the output voltage control.
Furthermore, by ascertaining that all roots of the characteristic equation lie in the left-hand s-plane for the 1+T, it achieves the complete stable time response. Fig. 8 shows the Bode plot of the closed-loop output impedance which is more damped than in the case of the open-loop in Fig. 6 (b).

4. Novel Systematical Modeling Approach

To confirm the effectiveness of the proposed approach to IBoFC modeling, a Bode plot of the control-to-output transfer function was obtained using a schematic-based PSIM simulation tool. The test conditions are summarized in Table 1. By comparing the simulation and the theoretical waveforms in Fig. 9, it becomes apparent that the frequency response plots are almost identical. It should be noted that the unexpected change above 100 kHz is caused by the switching frequency in the simulation setting.

To verify the theoretical operation and evaluate the performance of the proposed converter, a 100W IBoFC prototype was designed. An IRFB4227PBF MOSFET (VDS = 200 V, ID@25℃ = 65 A, RDS(ON) = 19.7 mΩ) from IR was used for the main switch (S1), UH10FT diodes (VRRM = 300 V, IF = 10 A, trr = 25 ns) from VISHAY were used for the diodes (D1, D2), and an IDH02SG120 SiC diode (VRRM = 1200 V, IF = 2 A) from Infineon was used for the diode (D3). For the transformer, a pair of ferrite cores (TDK, PC40EER40) was used, and 25 turns and 125 turns were wound for N1 and N2, respectively. Fig. 10 shows a photograph of the experimental setup, and the other experimental conditions are the same as in Table 1.

Fig. 11 shows the experimental frequency response of the IBoFC obtained using a Frequency Response Analyzer (Venable model 3120). The experimental result matches closely with the theoretical frequency response in Fig. 9 in the low frequency region under 1 kHz, while it shows a different trend in the high frequency region over 1 kHz due to the equivalent series resistance of the output electrolytic capacitor (Co). Since the primary concern for the relevant controller design is the low frequency region and the designed system bandwidth is 100 Hz, the difference in the high frequency region is not critical.

Fig. 12 shows the operational waveforms of the IBoFC at full load conditions. It can be seen that the boost inductor current flows in the DCM.

5. Conclusion

This paper proposed a novel systematic modeling
approach of the integrated single-stage power converter for dynamic analysis. A detailed modeling procedure was presented, specifically on a target of the IBoFC, as a typical example of the integrated power converter. The proposed approach simplifies the mathematical process of IBoFC modeling and provides straightforward full-order dynamic equations. The simulation and experimental results show the validity of the systematic modeling approach and the effectiveness of the voltage control based on the derived model.

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References


