Resonant Frequency Estimation of Reradiation Interference at MF from Power Transmission Lines Based on Generalized Resonance Theory

Tang Bo†, Chen Bin*, Zhao Zhibin**, Xiao Zheng*** and Wang Shuang*

Abstract – The resonant mechanism of reradiation interference (RRI) over 1.7 MHz from power transmission lines cannot be obtained from IEEE standards, which are based on researches of field intensity. Hence, the resonance is ignored in National Standards of protecting distance between UHV power lines and radio stations in China, which would result in an excessive redundancy of protecting distance. Therefore, based on the generalized resonance theory, we proposed the idea of applying model-based parameter estimation (MBPE) to estimate the generalized resonance frequency of electrically large scattering objects. We also deduced equation expressions of the generalized resonance frequency and its quality factor $Q$ in a lossy open electromagnetic system, i.e. an antenna-transmission line system in this paper. Taking the frequency band studied by IEEE and the frequency band over 1.7 MHz as object, we established three models of the RRI from transmission lines, namely the simplified line model, the tower line model considering cross arms and the line-surface mixed model. With the models, we calculated the scattering field of sampling points with equal intervals using method of moments, and then inferred expressions of Padé rational function. After calculating the zero-pole points of the Padé rational function, we eventually got the estimation of the RRI’s generalized resonant frequency. Our case studies indicate that the proposed estimation method is effective for predicting the generalized resonant frequency of RRI in medium frequency (MF, 0.3–3 MHz) band over 1.7 MHz, which expands the frequency band studied by IEEE.

Keywords: Reradiation interference from transmission lines, Resonant frequency, Generalized resonance theory, Model-based parameter estimation, Method of moments, Quality factor

1. Introduction

The smart grid in China, which has a core of UHV grid, is growing fast. As a result, the reradiation interference (RRI) from UHV transmission lines to nearby radio stations has become a major issue to the construction of UHV projects [1], and it urges to be solved. Currently, the related researches in China are focusing on setting National Standards that determines the protecting distances between UHV transmission lines and nearby radio stations [2-6].

However, the calculations of these protecting distances ignored interference resonance; instead, they took some certain frequencies discontinuously to calculate the interference level, which resulted in excessive redundancy of the protecting distances. Plus, applying protecting distance cannot deal with the interference between existed transmission lines and radio stations. Therefore, deeply understanding the reradiation resonant mechanism of transmission lines and estimating the resonant frequency (fr) have become a major topic of the electromagnetic compatibility between UHV lines and nearby radio stations recently.

In early stages of RRI researches, experiments and measurements on scaled models of transmission lines were quite common. The level and maximum values of interference were determined through observing the variation of spatial electric field intensity. By the 1980s, IEEE published the variation pattern of RRI’s induced current and electric field intensity based on antenna theories and estimation formulas of fr within 535–1,705 kHz [7]. However, these works based on electric field intensity cannot explain the RRI resonance that exists in frequency band over 1.7 MHz.

Tang Bo et al. examined the RRI from ±800 kV UHVDC transmission lines to AM broadcast stations (work at 536.5 kHz–26.1 MHz). They indicated that at frequencies over 1.7 MHz, RRI still has resonance, as well as periodic oscillation, and its amplitude tends to increase. The team also discussed the influences of tower numbers, grounding wires, conductors, and line spans on fr based on numerical calculation [8, 9]. In high frequency (HF, 3–30 MHz) band, the resonance of RRI is caused by the influence of transmission lines’ macroscopic structure on radio waves,
which is mainly determined by line span. However, the two works are still based on traditional methods that studying electric field intensity, so they can neither explain the generation of RRI resonance in HF band, nor predict resonant frequencies. Solving these two problems would require introducing other theories and analyzing methods.

From the perspective of energy balance of electromagnetic fields, Li Long et al. successfully turned the study of resonance in a multi-antenna system, i.e. an open electromagnetic field, into a study targeting at the resonance quality factor of a multi-port network based on generalized resonance theory and model-based parameter estimation (MBPE) [10-14]. In the study, MBPE is mainly used to construct the interpolating function directly associated with resonant frequencies and electromagnetic fields in the complex frequency domain. Ref. [10-14] presented acceptable estimations of multi-antenna system’s resonant frequency in near field region. Hence, if we take a transmission line in a wide space as a similar open electromagnetic system, we can introduce generalized resonance theory into studies of RRI from electrically large objects, namely transmission lines here, using the similar ways in [14].

In this paper, we estimate the resonant frequency of open electromagnetic systems composed of line antennas and transmission lines. The estimation is based on the function values (including the amplitude and phase of scattering fields), which is calculated by method of moments, of sampling frequency points and their zero-pole point characteristics. Further analysis indicates that the resonant frequencies estimated according to generalized resonance theory are valid and accurate, which means we can apply the theory to research the RRI from transmission lines.

2. Study of RRI Resonance Based on Field Intensity Analysis

2.1 Mechanism of RRI resonance and estimation of resonant frequency proposed in 1980s

Based on loop antenna theory and the results reported in [15-20], Trueman C.W. et al. explained the mechanism of RRI resonance in AM radio frequency band (535 kHz~1705 kHz) when towers are connected to grounding wires. Since the RRI level and the induced current in grounding wires have similar trends in frequency band below 1.7 MHz, they suggested that the induced current in grounding wires is the key factor determining the RRI resonance [16]. They also proposed that the induced current in grounding wires distributes as stationary waves at resonant frequencies. Trueman C.W. et al. explained the mechanism of RRI resonance in AM radio frequency band when towers are not connected to grounding wires as well, based on half-wave antenna theory and the results presented in [19]. Eventually, they proposed two resonant frequency mechanisms, namely the resonant frequency at integral multiples of wavelength and the resonant frequency at quarter wavelength.

1) When towers and grounding wires are not insulated, a grounding wire would link two adjacent towers. The three objects, along with their mirror images in the ground, form a loop antenna. When this loop antenna’s length equals 1.08 times of integral multiples of the wavelength, there will be RRI resonance. 1.08 is an empirical coefficient obtained from calculation and experiments [18].

2) When towers and grounding wires are linked by surge arresters, they are insulated. Then the grounding wires could be taken as conductors, and they have ignorable effect on RRI. In this circumstance, we can treat a tower in an alternating electromagnetic field as a line antenna which is perpendicular to the ground. According to half-wave antenna theory, when the height of this tower, i.e. the tower height, equals a quarter of the wavelength, there would be RRI resonance. Moreover, the tower’s electrical height is larger than its practical height by 15% if taking its cross arms into consideration [7].

2.2 RRI resonance in HF band

The line model of towers, which was adopted in the researches mentioned above, is too rough for calculating the RRI at higher frequencies. Tang Bo et al. proposed a line-surface mixing model that applies to HF band [6]. With this model, they calculated the protecting distance between the ±800 kV UHVDC transmission lines and AM broadcast stations (work at 536.5 kHz~26.1 MHz) [8, 9]. They indicated that at frequencies over 1.7 MHz, RRI still has resonance, as well as periodic oscillation, and its amplitude tends to increase.

To explain the RRI resonance in HF band, Tang Bo et al. studied the RRI level and the induced current in grounding wires at short-wave and higher frequencies [5]. At frequencies above 1.7 MHz, the induced current gets less corresponding to the extremes of RRI. The difference between the two increases with rising frequency. Therefore, when the frequency is higher than 1.7 MHz, the induced current in grounding wires is no longer a decisive factor to the RRI, and the law of the resonant frequency at integral multiples of the wavelength is no longer applicable.

3. Generalized Resonances in Open Electromagnetic Systems

3.1 Generalized resonances in a multi-antenna system

Currently, there are many in-depth studies of electromagnetic resonance in closed chamber and circuits, but relatively few ones of the resonances between antennas and
electrically large scattering objects in an open space with no explicit closed boundary.

In 2000s, several researchers from Xidian University in China defined the generalized resonance as the sudden rise of electromagnetic scattering at certain frequencies, based on their works on large electrical scattering objects including airplanes, ships, etc [10-14]. With in-depth studies and analyses of generalized resonance based on transmission line theory and generalized resonance cavity theory, they inferred generalized Foster theorem and the formulas for calculating the quality factor $Q$ and the preconditions of generalized resonance.

### 3.2 Generalized resonances in an antenna-transmission line system

Though the idea of generalized resonance proposed in [14] was describing the resonance for antennas in the near-field region of a multi-antenna system, the theories and the related conclusions are reasonable references for studying the RRI from transmission lines in an open space. Hence, from the perspective of the energy of an electromagnetic field and based on generalized resonance theory, we can turn the traditional way of analyzing electric field into the study of $Q$-value in an electromagnetic open system for understanding the RRI from transmission lines. That is to say, the value and appearance frequency of $Q$ could reflect the resonance feature of transmission lines.

### 4. RRI in Lossy Open Electromagnetic Systems and its Quality Factor

#### 4.1 Generalized resonant frequency and quality factor

As shown in Fig. 1, an electromagnetic open system composed of antennas and one transmission line can be regarded as a generalized closed system composed of surface $S_e$ and surfaces $S_i$. In this system, there are $N$ line antennas and one transmission line. The terminal voltage and terminal current of antenna $i$ are $U_i$ and $I_i$ respectively ($i=1, \ldots, N$).

The radiation of the antennas to infinity space and the joule heat on the transmission line are both losses to the system, so the system in Fig. 1 is a lossy electromagnetic open system. By introducing a complex resonant frequency $\hat{\omega}$ to replace the resonant frequency $\omega_0$, we consider the lossy system as a lossless system in form, where

$$s = j \hat{\omega} = \alpha + j\omega_0$$

$\alpha$ is the loss of the system.

Applying complex Poynting theorem in source-free regions to the space $V = \sum_{i=1}^{N} V_i$, we obtained the complex power balance equation for a generalized closed system, namely an antenna-transmission line system here.

$$\sum_{i=1}^{N} \frac{1}{2} U_i I_i^* = \Re \int \frac{1}{2}(\vec{E} \times \vec{H}^*) \cdot \hat{n} ds + \cdots + P_{\text{rad}} + 2\alpha (W_m + W_e) + \Re \int \frac{1}{2} \sigma \vec{E} \cdot \vec{E}^* dv$$

(2)

where $P_{\text{rad}}$ is the radiation loss from antennas to infinity space, $W_m$ and $W_e$ are the magnetic field energy and electric field energy in the space $V = \sum_{i=1}^{N} V_i$.

$$P_{\text{rad}} = \Re \int \frac{1}{2}(\vec{E} \times \vec{H}^*) \cdot \hat{n} ds$$

(3)

$$W_m = \Re \int -\sum_{i=1}^{N} \frac{1}{4} \mu \vec{H} \cdot \vec{H}^* dv$$

(4)

$$W_e = \Re \int -\sum_{i=1}^{N} \frac{1}{4} \varepsilon \vec{E} \cdot \vec{E}^* dv$$

(5)

Use $\Im \int \frac{1}{2}(\vec{E} \times \vec{H}^*) \cdot \hat{n} ds = 2\alpha (W_m - W_e)$ to stand for the electromagnetic energy exchanged on the surface $s$. Then there are $\vec{W}_m = W_m + W_e$ and $\vec{W}_e = W_e + W_e$. Combining with (2) there is

$$\sum_{i=1}^{N} \frac{1}{2} U_i I_i^* = \sum_{i=1}^{N} \frac{1}{2} I_i^* Z = \sum_{i=1}^{N} \frac{1}{2} U_i^* Y$$

$$= P_{\text{rad}} + 2\alpha (W_m + W_e) + \Re \int \frac{1}{2} \sigma \vec{E} \cdot \vec{E}^* dv$$

(6)

where $W_m$ and $W_e$ represent the magnetic field energy and the electric filed energy radiated out from the system respectively. $\vec{W}_m$ and $\vec{W}_e$ are the magnetic field energy and the electric filed energy in the system respectively.

According to [14], a generalized lossy open system will resonate when it fulfills (7) and (8).
where $P_L$ is the power loss of the system, $W$ is the electromagnetic energy in the system.

The antenna-transmission line system is a lossy network in nature (neither $P_L$ nor $\alpha$ is zero). When it has an RRI resonance, it loses very few energy compared to the massive electromagnetic energy it has stored. Therefore, in the system, the $\alpha$ according to the complex resonant frequency is small. Eq. (8) indicates the precondition of generalized resonance in an open space: the generalized electromagnetic energy in the system and on the outgoing boundary face of the Poynting vectors reaches a balance.

Moreover, a key factor that determines the resonant frequency and some other general characteristics of a resonant system is the system’s quality factor, $Q$. The common definition of $Q$-value for all kinds of resonate systems is

$$Q = \frac{\omega_0 W}{P_L}$$  \hspace{1cm} (9)

where $W$ is the time average of the energy in a resonant system, and $P_L$ is the energy loss per second in the system.

From (7) and (9), there is

$$\alpha = \frac{P_L}{2W} = \frac{\omega_0}{2Q}$$  \hspace{1cm} (10)

Then the equalized complex resonant frequency would be

$$s = j\omega = \frac{\omega_0}{2Q} + j\omega_0$$  \hspace{1cm} (11)

From (11), we can tell that the introduction of the equalized complex resonant frequency links the resonant frequency $\omega_0$ and the quality factor $Q$ of a resonant system. That is to say, knowing a resonant system’s complex resonant frequency $\omega$ equates with knowing the system’s $\omega_0$ and $Q$-value.

$Q$-value reflects not only a resonant system’s frequency selectivity, but also the generalized lossy feature of the system’s generalized resonance. Specifically, it shows the actual lossy feature inside the system (or the local area) including conductive or medium loss, as well as the energy leaking feature of the system (or the local area). According to the results in [14], for the system with a resonant $Q$-value large enough, there will be the resonance phenomena named “the strong peak field”, i.e. the field intensity reaches an extreme value. The greater $Q$-value indicates more energy stored in the system, and the stronger system resonance. Meanwhile the smaller $Q$-value indicates the greater energy loss of the system or the more energy loss from the space, which means less possibility of resonance in the system. Considering the definition of RRI, there is a positive correlation between $Q$-value and RRI resonance for an electromagnetic open system, that is to say. The intensity of RRI resonance can be reflected by $Q$-value effectively.

4.2 Calculating complex resonant frequency based on MBPE

According to Chapter 4.1, the resonant frequency and the quality factor $Q$ of a transmission line, i.e. an electrically large scattering system, could be obtained by calculating the system’s complex resonant frequency. There are some reports of constructing Padé rational function in the complex frequency domain of electromagnetic open systems including line antennas and radars based on MBPE [21-24]. In this way, the systems’ complex resonant frequency could be obtained through factorization.

However, there is no report about applying MBPE to calculate the complex resonant frequency of electrically large scattering objects yet. Since the transmission line excited by radio signals could be treated as an electromagnetic open system in a wide space, and it has similar physical characteristics to a radar cross section, we try to apply MBPE to calculate the complex resonant frequency of transmission lines.

MBPE uses low-order analytical formulas as the fitting models, in which the unknown coefficients are sampled from multi-point values.

According to the above calculation method in MBPE, when calculating the scattering field of a transmission line, we choose several frequencies within the frequency band under study as the sample points. Then the vector expressions of the scattering fields of the sample points shall be obtained through experimental measurements or numerical calculation.

Normally transmission lines distribute in areas with complex terrain conditions, and the actual towers are large, so it is hard to measure scattering fields in tests of real models. We use the method of moments to calculate the amplitude $|E(r)|$ and the phase $\phi(r)$ of the scattering field of the sample points.

According to the theories of time-varying electromagnetic fields, a line antenna driven by sine voltage source would radiate electromagnetic waves that induce sine scattering electromagnetic waves on the surface of the metal parts of a transmission line. The two kinds of electromagnetic waves shall have the same frequency. Therefore, we can express the transient value of the scattering field at any place by a cosine function.

$$E(r,t) = |E(r)|\cos(\omega t + \phi_{in}(r))$$  \hspace{1cm} (12)
where \( |E(r)| \) is a spatial function representing the amplitude of the function, \( \omega \) is the angular frequency, \( f \) is the frequency, and \( \omega = 2\pi f \). \( \phi^{\text{deg}}(r) \) is the initial phase of the cosine function, and it might be a space function.

According to complex vector theory, we can describe the scattering field with a complex number which is independent of time.

\[
\dot{E}(r) = |E(r)| e^{i\omega t} \tag{13}
\]

Using Euler formula \((e^{i\theta} = \cos\theta + j\sin\theta)\), Eq. (13) is transformed into

\[
\dot{E}(r) = |E(r)| \left[ \cos(\phi^{\text{rad}}) + j\sin(\phi^{\text{rad}}) \right] \tag{14}
\]

where \( \phi^{\text{deg}} \) is phase of the scattering field in degree, \( \phi^{\text{rad}} \) is the radian value of \( \phi^{\text{deg}} \), and \( \phi^{\text{deg}} = \pi (90 + \phi^{\text{deg}})/180 \).

After setting the position of field point \( r \), \(|E(r)|\) and \( \phi^{\text{deg}}(r) \) are functions of only frequency. We use the low-order Padé rational function as the interpolating function, \( \dot{E} \) as the dependent variable, and the complex frequency \( s \) corresponding to the sample frequency as the independent variable, to construct an interpolating function for the frequency response of the scattering field of a transmission line.

\[
\dot{E}(s) = \frac{P_L(s)}{Q_M(s)} = \frac{b_0 + b_1 s + b_2 s^2 + \cdots + b_L s^L}{a_0 + a_1 s + a_2 s^2 + \cdots + a_M s^M} \tag{15}
\]

where \( L \) and \( M \) are the highest orders of the two multinomial \( P_L(s) \) and \( Q_M(s) \), \( s \) is the complex frequency \((s = j\omega)\), \( b_l \) \((l = 0, 1, \ldots, L)\) and \( a_m \) \((m = 0, 1, \ldots, M)\) are the coefficients of the Padé rational function’s numerator and denominator to be solved. There is no common factor between \( P_L(s) \) and \( Q_M(s) \), and \( Q_L(s) \neq 0 \).

If we reduce \( a_m \) or \( b_l \) to 1, there will be \( P = L + M + 1 \) coefficients to be solved. Hence using the method of moments, we obtain \( P \) sample points in the frequency range under study, namely \((s_i, \dot{E}(s_i)), i = 1, 2, \ldots, P \). To deduce the equation of Padé interpolating function, we set \( a_m = 1 \). Then the matrix form of Eq. (15) is

\[
A_X = B \quad (i = 1, 2, \ldots, P) \tag{16}
\]

\[
A_i = \begin{bmatrix}
1 s_{i} \cdots s_{i}^L - \dot{E}(s_i) - \dot{E}(s_i) s_{i} \cdots \dot{E}(s_i) s_{i}^{M-1} \\
1 s_{i}^2 \cdots s_{i}^L - \dot{E}(s_i) - \dot{E}(s_i) s_{i} \cdots \dot{E}(s_i) s_{i}^{M-2} \\
\vdots \vdots \cdots \cdots \vdots \\
1 s_{i}^p \cdots s_{i}^L - \dot{E}(s_i) - \dot{E}(s_i) s_{i} \cdots \dot{E}(s_i) s_{i}^{M-p} \\
1 s_{i}^p \cdots s_{i}^p - \dot{E}(s_i) - \dot{E}(s_i) s_{i} \cdots \dot{E}(s_i) s_{i}^{M-p} \\
\dot{E}(s_i) s_{i}^p - \dot{E}(s_i) - \dot{E}(s_i) s_{i} \cdots \dot{E}(s_i) s_{i}^{M-p} \\
\end{bmatrix}
\]

\[
X_i = [b_0, b_1, \ldots, b_L, a_0, \ldots, a_{M-1}]^T \tag{17}
\]

\[
B_i = [\dot{E}(s_i) s_{i}^p \dot{E}(s_i) s_{i} \cdots \dot{E}(s_i) s_{i}^{M-p} - \dot{E}(s_i) - \dot{E}(s_i) s_{i} \cdots \dot{E}(s_i) s_{i}^{M-p}]
\]

Eventually we can get the coefficients \( b_l \) \((l = 0, 1, \ldots, L)\) and \( a_m \) \((m = 0, 1, \ldots, M)\) by solving the (17)–(19) of \( P \) elements, and there is

\[
\dot{E}(s) = k \frac{(s - r_0)(s - r_1) \cdots (s - r_L)}{(s - s_1)(s - s_2) \cdots (s - s_M)} k \prod_{i=1}^{L} \frac{s - r_i}{s - s_i} \tag{20}
\]

where \( r_i \) \((i = 0, 1, \ldots, L)\) and \( s_m \) \((m = 0, 1, \ldots, M)\) are complex zero points and complex pole points, \( k \) is a scale factor.

The orders of the numerator and denominator of the Padé rational function are empirically chosen according to the complexity of the problem to be solved. According to uniform approximation theory, when \( L = M \) or \(|L - M| = 1 \), the result of a rational function interpolation has the smallest deviation from the practical results.

5. Case studies and analyses

5.1 RRI resonance in MF

5.1.1 Towers linked by grounding wires

According to [15-20], conventionally in 535~1,705 kHz, a tower is equivalent to a line antenna of 2.13~4.88 m. Trueman et al. established the numerical model of a 500 kV double-circuit transmission line with V1S towers [20], as shown in Fig. 2. There are nine towers in the model. The towers and the grounding wire are equalized to line models of 3.51 m and 0.71 m in radius. To ensure the omnidirectional signal transmitting ability of the broadcast antenna, the RRI from the transmission lines in the direction toward the MF broadcast antenna is the main target. The broadcast antenna is the excitation source in the model. It is 195 m high, and 448 m far from the central line of the transmission line. Its supply voltage is 1 V, and its feeding terminal is on the ground. They calculated the RRI in the far field.

Based on generalized resonance theory and MBPE, we calculate the complex resonant frequency of the open electromagnetic system shown in Fig. 2, and consequently obtain the resonant frequency and quality factor \( Q \) through (11).

Firstly, we establish a Padé rational function, which is
similar to (15), based on MBPE. As mentioned in Chapter 4.2, we set both the orders of the rational function’s numerator and denominator to be 4. That is to say, \( L=M=4 \) and there are nine coefficients to be solved. Hence, it needs nine frequencies at equal intervals in the medium frequency band. Here we choose 0.2 MHz as the first frequency, and the frequency interval between every two adjacent sample frequencies is 0.395 MHz. The model in frequency, and the frequency interval between every two frequency band. Here we choose 0.2 MHz as the first frequency, and the frequency interval between every two adjacent sample frequencies is 0.395 MHz. The model in Fig. 2 is segmented into 15 m parts. Then the amplitude and phase of the scattering field at the observing points at the nine sample frequencies are calculated. Based on the results, the nine coefficients of the Padé rational function are calculated using (16). Then we obtain the complex zero points and complex pole points of the Padé rational function from its factorization. The complex pole points are the complex resonant frequencies characterizing the electromagnetic system.

Using (11), the resonant frequencies and the quality factors corresponding to the complex resonant frequencies are calculated, as shown in Table 1. It is mentioned in Chapter 4.1 that, in a lossy open electromagnetic system, a resonant frequency would correspond to a relatively large quality factor. Table 1 indicates that the most intense resonance occurs at 1.2028 MHz along with the highest \( Q \)-value. Meanwhile, the \( Q \)-value at 1.4507 MHz is overly different from that of the other frequencies (pole points marked by *) compared to itself, therefore the corresponding resonance was neglected.

Truemen C.W. et al. built a 1/600 scaled model of the actual transmission line corresponding to the numerical model in Fig. 2 [20]. They obtained the resonant frequencies in medium frequency range corresponding to integral multiples of the wavelength through experiments, namely 0.434 MHz, 0.850 MHz and 1.280 MHz.

The length of the loop antenna corresponding to the model in Fig. 2 is 751.6 m. Using the resonant frequency estimation method mentioned in Chapter 2.1, we calculate the resonant frequencies of the circuits of one, two and three times of the wavelength long. Taking the measured values as true values, we calculate the deviations of the calculated values using the method proposed by Trueman C.W. and the generalized resonance method, as shown in Table 2.

### 5.1.2 Towers insulated from grounding wires

When the towers are insulated from the grounding wire, the grounding wire equates with a conductor, and it barely affects the RRI [19]. Trueman C.W. et al. built the tower model of a V1S tower on a 500 kV double-circuit transmission line, taking the influence of the cross arms on the electric tower height into consideration, as shown in Fig. 3. The model is a single tower, and it equates the angle steels with cylinders of 0.01 m in radius. Taking the vertically polarized plane electromagnetic wave in 0.6~2 MHz as the excitation source, and setting the electric field intensity to be 1 V/m, they studied the RRI at one mile distance far from the tower.

We calculated the complex resonant frequency of the system in Fig. 3. 0.6 MHz is the first sample frequency, and every two sample frequencies have interval of 0.168 MHz. Segmenting the model in units of 0.1 times of the wave length, we calculated the scattering field at each sample frequency using the method of moments, and consequently obtained the nine-order Padé rational function in the complex frequency domain. Then the complex pole points and complex zero points of the Padé rational function is obtained from its factorization, as shown in Table 3. The \( Q \)-value of 1.0752 MHz is obviously larger than the others, which means the resonance of the system shown in Fig. 3 occurs at 1.0752 MHz.

Trueman C.W. et al. measured the resonant frequency corresponding to the quarter wavelength through scaled experiments, and it was 1.105 MHz [19]. Using the estimation method mentioned in Chapter 2.1, the estimated number of this resonant frequency is 1.2033 MHz. Taking

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**Table 1.** Pole-zero points, resonant frequency and quality factor of Padé rational function

<table>
<thead>
<tr>
<th>( E[3,3] )</th>
<th>Zero points</th>
<th>Pole points</th>
<th>Resonant frequency (MHz)</th>
<th>Quality factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.0139 + 1.4288i</td>
<td>0.0127 + 0.4129i*</td>
<td>0.4129</td>
<td>16.2559</td>
</tr>
<tr>
<td>2</td>
<td>0.2528 + 1.3711i</td>
<td>0.0416 + 0.7827i*</td>
<td>0.7827</td>
<td>9.4075</td>
</tr>
<tr>
<td>3</td>
<td>-0.0539 + 0.7857i</td>
<td>0.0194 + 1.2028i*</td>
<td>1.2028</td>
<td>31.0000</td>
</tr>
<tr>
<td>4</td>
<td>0.1302 + 0.1407i</td>
<td>-0.1302 + 1.4507i</td>
<td>1.4507</td>
<td>5.5710</td>
</tr>
</tbody>
</table>

**Table 2.** Comparison of Trueman’s, generalized resonance predictive values and experimental measurements

<table>
<thead>
<tr>
<th>Measurement from scaled model</th>
<th>Trueman’s estimation</th>
<th>Generalized resonance estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>Deviation</td>
<td>Value</td>
</tr>
<tr>
<td>0.434</td>
<td>-0.003</td>
<td>0.4129</td>
</tr>
<tr>
<td>0.850</td>
<td>0.012</td>
<td>0.7827</td>
</tr>
<tr>
<td>1.280</td>
<td>0.013</td>
<td>1.2028</td>
</tr>
</tbody>
</table>

**Fig. 3.** Power transmission line model built by C.W. Truemen in 1989
the measured values as true values, we calculate the deviations of the calculated values using the method proposed by Trueman and the generalized resonance method, as shown in Table 4.

5.2 RRI at frequencies over 1.7 MHz in MF band

The ratio of the tower details to the excitation wavelength increases with the excitation frequency. Eventually, the line model used by former studies is no longer applicable above a certain frequency. Therefore, the related calculations require more exquisite model that takes the truss structure of the tower into account. Tang Bo et al. established a line-surface mixing model for calculating the RRI from the ±800 kV Xiangjiaba-Shanghai UHVDC transmission line, as shown in Fig. 4. The model has five ZP30101 towers, which are 63 m tall, and having cross arms of 41.2 m and spans of 500 m. The towers are linked by two grounding wires.

Since the antennas of radio stations vary in type, size and model, the RRI from transmission lines also varies. Therefore, we cannot use a specific antenna to excite the simulative model; instead, we excite the model with plane electromagnetic waves incoming from different stations at infinite distance. Considering the tower is vertical to the ground, here we take the most severe condition, i.e., use a vertically polarized plane wave excitation.

For comparison, we calculated the RRI at the position (0, 2000, 2) in the MF band of 1.7~3.0 MHz [25]. The excitation is vertically polarized plane wave, and the exciting electric field intensity is 1 V/m. We calculated the complex resonant frequency of the system in Fig. 4. 1.7 MHz is the first sample frequency, and every two sample frequencies have interval of 0.156 MHz. Segmenting the model in units of 0.1 times of the wave length, we calculated the scattering field at each sample frequency using the method of moments, and consequently obtained the nine-order Padé rational function in the complex frequency domain. Then the complex pole points and complex zero points of the Padé rational function is obtained from its factorization, as shown in Table 5. According to Table 5, it is predicted that obvious resonances occur at 2.0324MHz. In addition, since 2.4947 MHz has a relatively larger $Q$-value, it could be considered as a resonant frequency.

According to Chapter 2.2, when the excitation frequency is over 1.7 MHz, the resonant frequency mechanism of integral multiples of the wavelength is no longer applicable. With frequency intervals of 0.01 MHz, we carry out frequency-sweeping calculation of RRI of the model of Fig. 4 under the frequencies included in the target range using the method of moments. The results are shown in Fig. 5. There are two obvious resonant frequencies, namely 2.040 MHz and 2.558 MHz. With these two as the true values, we get the deviations of the two estimated resonant frequencies, namely -0.0076 MHz and -0.0633 MHz respectively. According to the peak RRI in Fig. 5, the intensity of RRI resonance at 2.040 MHz is stronger than that at 2.558 MHz, and the corresponding $Q$-value of 2.0324 MHz is larger than that of 2.4947 MHz. This supports the positive correlation between $Q$-value and RRI resonance proposed mentioned in [14].

5.3 Analyses of the results

5.3.1 Estimation accuracy

According to Table 2 and Table 4, the estimations based on generalized resonance theory has smaller deviations than that of integral multiples and quarter wavelengths from the measured value obtained from scaled experiments.
Hence, generalized resonance theory is applicable for analyzing the resonance in lossy open electromagnetic systems, and it improves the prediction of resonant frequencies in accuracy, compared to conventional methods.

### 5.3.2 Estimating resonance intensity

According to Chapter 4.1, the quality factor reflects not only a resonant system’s frequency selectivity, but also the generalized lossy feature of the system’s generalized resonance. If the quality factor according to a pole point is small, there is no RRI resonance. Otherwise, RRI resonance would appear along with resonance peaks. In Table 5, the quality factor at is the largest 2.0324 MHz, and it is much smaller at 2.4974 MHz. Fig. 5 directly shows the resonance peaks at the two resonant frequencies. The peaks get smaller with decreasing quality factor.

### 5.3.3 Applicable frequency range

According to Chapter 2.2, when the excitation frequency is over 1.7 MHz, the induced current in the grounding wire is no longer the factor which determines the RRI resonance, and the RRI resonant frequency does not fulfill the integral multiples of the wavelength anymore. In the calculation of Chapter 5.2, we successfully estimated the resonant frequencies in 1.7-3.0 MHz of the system excited by vertically polarized plane waves, based on generalized resonance theory and using a line-surface mixing model of the system. Though the estimated values have deviations, they meet the requirement of ±100 kHz resonance bandwidth in [7]. Hence, generalized resonance theory provides an efficient way to expand the predictable frequency range of RRI resonances.

### 5.4 Error analysis

#### 5.4.1 Error caused by the simplification of physics

The induced current at any source point of a transmission line is not only influenced by the point’s location and the excitation electromagnetic wave’s frequency, but also affected by the scattering fields from the other surrounding source points. Hence, when calculating the scattering field of a transmission line in the full frequency bands, the induced current \( J \) should be a function of the frequency \( f \) and the source point position \( r' \). Specifically, the induced current \( J \) at the source point \( r'_j \) would be

\[
J(f, r'_j) = \sum_{i=1}^{N} E_i(R_{ij}) \exp[-j2\pi f t_i(r'_j)]
\]  

(21)

where \( E_i(R_{ij}) \) is the scattering field at \( r'_j \) from the induced current at the source point \( r' \). \( R_{ij} = r'_j - r'_i \), and \( t_i \) is the delay that the scattering field from \( r'_i \) takes to get to \( r'_j \). Therefore, the induced current varies with the excitation frequency. However, the application of MBPE requires \( E(R_{ij}) \) in (21) to be comparatively constant for the excitation frequency. It means the MBPE cannot fully reflect the physics law related to the induced current, which would result in certain error.

#### 5.4.2 Error exists in experimental measurements

The experiments used the scale model of transmission line. The scale model ignores all kinds of accessories and fittings including the clamps of grounding wires and conductors, so it has certain difference from its actual counterpart. Moreover, the scaled model is designed to be 1/600 of the actual transmission line in the scale while having electromagnetic wave frequencies and model conductivity 600 times as large as that of the actual line. However, it is nearly impossible to ensure the precise model conductivity, which would also result in errors.

#### 5.4.3 Error exists in simulative calculations

The established numerical model took the ground as ideal conductor, and it assumed the towers are ideally grounded. Because of these idealizations, the model only simulates the actual RRI from transmission lines incompletely. In fact, the induced current distributes continuously along the whole transmission line. Our simulation disperses the induced current using the method of moments, which removes the continuity of the induced current and induces the truncation error. Plus the simulation conducted in computers will always have round-off errors in each step of the calculation due to the limit of data length.

### 6. Conclusion

1) Based on generalized resonance theory and MBPE, we studied the RRI from transmission lines from the perspective of the energy of an electromagnetic field. In this way, we bypass the problem that the induced current’s influence on the intensity of scattering field varies with frequency, and realize the estimation of RRI resonant frequency in the whole MF range. It is an
expansion of the former researches in this area. According to the three case studies in this paper, the estimation of the resonant frequency has deviation of no more than ±100 kHz from the experimentally measured results.

2) The quality factor of resonance is greatly related to the extremes of RRI, so it characterized the resonance intensity of an open electromagnetic system. Specifically, the value of a quality factor and its corresponding frequency reflects the resonance feature of a transmission line. However, the functional relationship between \( Q \)-value and RRI value has not been obtained, i.e., we can’t calculate RRI value using \( Q \)-value. Further efforts are needed in this issue. In the meantime, we would keep working on the estimation of RRI from transmission lines in HF band.

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