Transformer Winding Modeling based on Multi-Conductor Transmission Line Model for Partial Discharge Study

Seyed MohammadHassan Hosseini† and Peyman Rezaei Baravati*

Abstract – To study and locate partial discharge(PD) and analyze the transient state of power transformer, there is a need for a high frequency model of transformer winding and calculation of its parameters. Due to the high frequency nature of partial discharge phenomenon, there is a need for an accurate model for this frequency range. To attain this goal, a Multi-Conductor Transmission Line (MTL) model is used in this paper for modeling this transformer winding. In order that the MTL model can properly simulate the transformer behavior within a frequency range it is required that its parameters be accurately calculated. In this paper, all the basic parameters of this model are calculated by the use of Finite Element Method (FEM) for a 20kV winding of a distribution transformer. The comparison of the results obtained from this model with the obtained shape of the waves by the application of PD pulse to the winding in laboratory environment shows the validity and accuracy of this model.

Keywords: Partial Discharge, Transformer, Transfer Functions, Finite Element Analysis, Multi-Conductor Transmission Line (MTL)

1. Introduction

With regard to the importance of transformers to be in circuit in power systems, their maintenance is very important. On the other hand, the insulation breakage of power transformers is of their major problems resulting from partial discharge gradually destroying the insulation. By early detection of partial discharge, a bulk of the repair costs is reduced [1]. The major methods for partial discharge locating can be divided into electric and non-electric methods. Most of the studies have been done on electric methods. Detection and locating of partial discharge in this method is based on obtaining the intended signals at the transformer terminals. By analyzing the recorded signals, it is possible to find out the exact place of partial discharge (PD) in the winding. For locating the partial discharge by electric method there is a need for a high frequency model of transformer winding. Many research activities have been performed on transformer winding modeling for the study and analysis of the propagation of PD signals for their location. With regard to these studies, two major methods which have been used can be proposed for the winding modeling: winding modeling by MTL model [2], and detailed winding model [3].

In the detailed model, called internal model or ladder RLC, the smallest space in a winding of a turn or a group of turns such as a disc or a pair of discs is modeled by an RLC circuit and the collection of the winding is in the form of a ladder network. Each element in the network has a capacitor relative to the ground and the inductances have also mutual induction. It is difficult in this method to estimate the circuit parameters with regard to the data related to the geometrical dimensions of transformer and the materials employed [4].

Contrary to the above method, the distributed parameters of transmission line are used for winding model in the MTL modeling method. Due to the presence of inductances and capacitors in transient states, traveling waves are created which are propagated at different speeds inside the winding and based on the total effects of these transient waves, a state of resonance is created. Based on the theory of multi-conductor lines, the winding modeling is in the form of transmission lines which are parallel in positioning, but are in series electrically [5, 6]. In this modeling, each winding unit is considered as one transmission line with distributed parameters and their collection forms a multi-conductor transmission system. This method is also applicable for electric machines, transformers, and cables [7].

In this paper, the model of MTL is used for the simulation of transformer winding in the study of high frequency phenomenon effects. A new solution approach to obtain the MTL model parameters by the use of finite element method (FEM) is also presented.

This paper is organized as follows: MTL model is introduced in the second part and the manner of obtaining the electrical parameters is explained. In the third part, a new method is explained for the manner of obtaining the electrical parameters of the model through finite element method (FEM). Then, a comparison is made in the fourth
part between the model simulation results and the laboratory results to assess and validate the model, and at the end the final conclusion is presented.

2. Multi-Conductor Transmission Line Model (MTL)

In the study of the partial discharge along the winding, the transformer winding is considered as a single input multiple output system (SIMO). Its input is the PD signal and its outputs are the measurement current signals of the terminals. Fig. 1 shows the transformer winding with n turns modeled by MTL method. Each turn of the transformer winding is modeled as one single-conductor transmission line in this model [8].

In Fig. 1 when each turn of the winding is expressed by one transmission line, the propagation phenomenon in the transformer winding can be expressed through telegraphic equations in time domain:

\[
\frac{\partial [u(x,t)]}{\partial x} = - \left( R \left[ [i(x,t)] + L \frac{\partial [i(x,t)]}{\partial t} \right) \right.
\]

(1)

\[
\frac{\partial [i(x,t)]}{\partial x} = - \left( \left[ G \left[ [u(x,t)] + C \frac{\partial [u(x,t)]}{\partial t} \right) \right) \right.
\]

(2)

Where \( u(x,t) \) and \( i(x,t) \) are the voltage and current vectors at position \( x \) along the line; \( R, L, G \) and \( C \) are the per unit length resistance, inductance, conductance and capacitance matrix, respectively.

Due to the symmetry of electrical parameters, the telegraphic equations in frequency domain are as follows:

\[
\frac{d^2 [\tilde{u}(x)]}{dx^2} = [Z][Y][\tilde{u}(x)] = [P]^2[\tilde{u}(x)] \quad (3)
\]

\[
\frac{d^2 [\tilde{i}(x)]}{dx^2} = [Y][Z][\tilde{i}(x)] = ([P]^2) \quad [\tilde{i}(x)] \quad (4)
\]

Where \( \tilde{u}(x) \) and \( \tilde{i}(x) \) are the phasor voltage and current vectors; \( [P]^2 = [Z][Y], ([P]^2) = [Y][Z], [Z] = R + j \cdot 2\pi f \cdot L \) and \( [Y] = [G] + j \cdot 2\pi f \cdot C \). \([Z] \) and \([Y] \) are impedance and admittance matrices of the model, \( f \) is the frequency.

By the use of this model based on the theory of traveling waves, the calculation of the voltage and transient current is possible through all the points along the winding.

Voltage vectors and current at \( x \) distance from the beginning of the line can be obtained by Eqs. (3) and (4) as follows:

\[
[\tilde{U}(x)] = \exp((-P)x) \cdot [\tilde{U}_1] + \exp((-P)x) \cdot [\tilde{U}_f] \quad (5)
\]

\[
[\tilde{I}(x)] = Y_0 \left[ \exp((-P)x) \cdot [\tilde{U}_1] - \exp((-P)x) \cdot [\tilde{U}_f] \right] \quad (6)
\]

Where, \( \tilde{U}_1 \) and \( \tilde{U}_f \) are the voltage vectors with regard to the boundary conditions, \( [Y_0] = [Y][P]^{-1} \) of model specification admittance matrix and \([P] \) Product of matrices \( Z \) and \( Y \).

By the use of boundary conditions which are as follows:

\[
I_h(i) = -I_t(i + 1) \quad i = 1, 2, ..., n - 1 \quad (7)
\]

\[
u_h(i) = u_t(i + 1) \quad i = 1, 2, ..., n - 1 \quad (8)
\]

Where \( u_h(i) \) and \( I_h(i) \) are sending end voltage and current of \( i \)-th transmission line and \( u_t(i) \) and \( I_t(i) \) are its receiving end voltage and current. If PD pulse is applied to \( K \)-th turn of the winding, Eq. (7) for \( i=k-1 \) is corrected as follows:

\[
I_t(k - 1) + I_h(k) = I_{PD} \quad (9)
\]

Where \( I_{PD} \) is the PD pulse current. By the application of the boundary conditions, the currents of the sending end and receiving end can be expressed in terms of voltage vectors:

\[
[\begin{bmatrix}
I_{h}(1) \\
0 \\
. \\
0 \\
I_{PD} \\
. \\
. \\
I_{t}(n)
\end{bmatrix}] = [Y]^{-1} \quad [\begin{bmatrix}
\tilde{U}_{h}(1) \\
\tilde{U}_{h}(2) \\
. \\
\tilde{U}_{h}(k) \\
. \\
. \\
\tilde{U}_{t}(n)
\end{bmatrix}] \quad (10)
\]

By inverting matrix \([Y]\), the transfer function matrix \([T]\) of transformer will be as follows:

\[
[\begin{bmatrix}
I_{h}(1) \\
\tilde{U}_{h}(2) \\
\tilde{U}_{h}(3) \\
. \\
\tilde{U}_{h}(k) \\
. \\
\tilde{U}_{t}(n)
\end{bmatrix}] = [T] = [Y]^{-1} \quad [\begin{bmatrix}
\tilde{U}_{h}(1) \\
\tilde{U}_{h}(2) \\
. \\
\tilde{U}_{h}(k) \\
. \\
\tilde{U}_{t}(n)
\end{bmatrix}] \quad (11)
\]
The transformer bushing can be included in the equations by \( C_0 \) capacitor connected to the beginning of the line.

After solving the stated equations, it is possible to calculate the current equation produced due to the occurrence of PD at the head and at the end of the winding. By considering the above conditions, the transfer functions for the location of PD occurrence toward the winding phase (\( TF_p \)) and up to the neutral end of the winding (\( TF_n \)) is as follows [9]:

\[
TF_p = \frac{i_p(1)}{i_{PD}} = \frac{\tau(N,K) - \tau(N,1) - \tau(1,N) + \tau(N,1)}{\tau(N,N) + \frac{\tau(N,K) - \tau(N,1) - \tau(1,N) + \tau(N,1)}{2\pi f C_B \tau(N,K)}}
\]

\[
TF_N = \frac{i_p(n)}{i_{PD}} = \frac{\tau(N,1) - 2\pi f C_B \tau(N,K)}{\tau(N,N) + 2\pi f C_B}
\]

Where, in Eqs. (12) and (13) \( N=\pi+1 \) and \( n \) is the number of the conductors in the model.

### 2.1 Calculation of the model parameters

#### A. Capacity matrix

Calculation of the capacitor capacity plays a significant role in the accuracy of this model. The capacitor capacity between each turn of the winding and the next turn is obtained as follows [10]:

\[
c = \frac{\varepsilon_0 \varepsilon_p \rho d_{wh}}{\sigma}
\]

Where, \( \varepsilon_0 \) is vacuum dielectric constant and \( \varepsilon_p \) is the relative dielectric coefficient of the paper and \( \rho \) is the thickness of the paper at the two sides of the conductor while, \( d_{wh} \) is the average diameter of the winding and \( h \) is its height of conductor.

#### B. Inductance matrix

Self-inductance for each turn of the winding and its mutual inductances form the inductance matrix with other turns of the winding. Magnetic flux penetration inside the iron core can be ignored in high frequencies assuming the winding with an air core. In mutual inductance calculation we have:

\[
L_nC_0 = C_0L_n = \frac{1}{\nu} E
\]

Where, \( L_n \) is the mutual matrix; \( C_0 \) is the capacity matrix in vacuum medium; \( \nu = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \) is the propagation speed of electromagnetic waves in vacuum; \( \varepsilon_0 \) is vacuum dielectric constant; \( \mu_0 \) is permeability of the vacuum and \( E \) is the unit matrix.

With regard to the skin effect, the self-inductance at high frequencies is calculated as follows:

\[
L_I = \frac{R_s}{2\pi f}
\]

Where, in Eq. (16) \( R_s \) is the resistance due to the skin effect, and \( f \) is the frequency. The whole inductance is calculated by Eq. (17):

\[
L = L_n + L_I
\]

#### C. Resistance calculation

The resistance equation of the conductor length unit considering the skin effect can be represented as follows:

\[
R_s = \frac{1}{2} \frac{\pi f \mu}{\sigma}
\]

Where, \( d_1 \) and \( d_2 \) are the width dimensions of a rectangular conductor; \( \mu \) is the conductor magnetic permeability coefficient; \( \sigma \) is the electric conductivity capability; and \( f \) is the frequency.

### 3. Calculation of Parameters by Finite Elements Method (FEM)

The Finite Elements Method (FEM) is a numerical method used for approximate solution of partial differential equations and also for the solution of integrals. The finite elements method as an accurate and valid method has great applications in simulating the behavior of transformers in different conditions. Of these applications, it can be referred to the calculation of leaking fluxes and electromagnetic forces. By the use of FEM method, the circuit parameters equal to winding are calculable by considering the details of the transformer structure. In this paper, ANSOFT MAXWELL software, which solves problems on the basis of the analysis of the elements, is employed to obtain the winding parameters.

#### 3.1 Calculation of capacity matrix

Based on electromagnetic laws, the total stored electric energy \( W \) in a space with a volume of \( V \) can be obtained by the following Eq. [11].

\[
W = \frac{1}{2} \sum_{i=1}^{n} \left( C_{ii} U_i^2 + \sum_{j \neq i} C_{ij} U_i U_j \right)
\]

Where, \( U_i \) and \( U_j \) are the voltage relative to conducting ground \( i \) and \( j \); \( C_{ii} \) the self-capacitor, and \( C_{ij} \) is the mutual capacitor capacity, respectively. The energy method is a method by which the capacity capacitor is calculated. To calculate the coefficients in this method, \( C_{ii} \) is applied to
the desired constant voltage in the conductor and the remaining voltage of conductors is equal to zero and the energy is calculated in each of the three states. Then, the capacitor capacities are simultaneously obtained. In this state, as it is shown in Fig. 3 the total energy is calculated as it was mentioned above. The diametric elements of the matrix \( C \) can be obtained by the following equation:

\[
C_{ii} = \int_{V} D_{i} E_{i} dV = \frac{2W}{\mu_{0}}
\]

Where, \( D_{i} \) is the density of the generated electric flux in conductor \( i \) when a 1-volt voltage is applied to the conductor; and \( E_{i} \) is the intensity of the generated electric field in conductor \( j \) when conductor \( j \) voltage is 1. To calculate the \( C_{ij} \) coefficients, constant voltages are applied to conductors \( i \) and \( j \) and the remaining conductors are kept at zero voltage. Like the calculation of \( C_{ii} \) by calculating the stored electric energy in space, the non-diametric elements of \( C \) matrix are obtained. The major non-diameter elements of \( C \) matrix can be obtained as follows:

\[
C_{ij} = \int_{V} D_{i} E_{j} dV = \frac{2W}{\mu_{0}}
\]

### 3.2 Calculation of inductance matrix

The stored energy in the magnetic field which couples two conductors is obtained as follows:

\[
\frac{1}{2} \mu_{0} I^2 = \frac{1}{2} \int_{\Omega} B_{i} H_{i} d\Omega
\]

Where, \( W_{ij} \) is the stored energy in the magnetic field of \( i \) and \( j \) couplers; \( I \) is the \( i \) conductor current; \( B_{i} \) is the density of the magnetic field resulting from the passage of 1-ampere current through \( i \) conductor; and \( H_{i} \) is the magnetic field resulting from the passage of 1-ampere current through \( j \) conductor. To calculate the inductance matrix at each stage, by applying the current to the conductors and considering other conductors as zero, energy (W) is calculated and diametric and non-diametric elements of \( L \) matrix are obtained. The calculated values of inductance for different turns of winding are shown in Figs. 6 to 8.

### 3.3 Calculation of resistance matrix

To calculate the resistance, we can first calculate ohms losses in a conductor. In fact, by the use of FEM method and solving the Maxwell equations, it is possible to obtain the field around every conductor and after calculating the current density by the use of equation \( J = \nabla \times H \), the losses generated in the conductor can be calculated. The generated losses in the conductor are equal to:

\[
P = \frac{1}{2\pi} \int_{S} J \cdot J' ds
\]

Table 1. Specifications of tested winding

<table>
<thead>
<tr>
<th>Transformer 20kV-winding parameters</th>
<th>Layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of winding turns</td>
<td>38 turns</td>
</tr>
<tr>
<td>Diameter of winding</td>
<td>260 mm</td>
</tr>
<tr>
<td>Height of winding</td>
<td>160 mm</td>
</tr>
<tr>
<td>Width of conductor</td>
<td>8 mm</td>
</tr>
<tr>
<td>Height of conductor</td>
<td>2 mm</td>
</tr>
</tbody>
</table>

Simulator considers the peak of flowing current in each conductor equal to one ampere, so the resistance is equal to 2P. To calculate inductance and resistance in MHz range, the skin effect should be considered in calculations.

Fig. 9 shows the manner of resistance size change in terms of turns for the 20th turn.

The specifications of the tested transformer winding depicted in Fig. 2 is presented in Table 1.
3.4 Modeling results

Magnetic results obtained from 20 kV transformer winding modeling are shown in Figs 3 to 5.

In order to study the amount of accuracy of presented parameters, the measured and calculated results are compared with one another. The measured and calculated magnitude of transfer function of turn 20 compared to the
beginning of the winding has been shown in Fig. 10. The blue diagram shows the transfer function magnitude of the transformer winding obtained in laboratory, and the red diagram illustrates the results of simulation of transfer function obtained from parameters.

Some of the calculated parameters for the winding in the experiment using analytical relations and finite elements method (FEM) have been presented in Table 2.

### 4. Study of the Model Validity

The distribution 20kV transformer winding was modeled and simulated by using the theory of MTL model and with the help of MATLAB software, and the parameters of this model were calculated by the Finite Elements Method in this paper. Having the quantities of transfer functions and pulse supposition for the partial discharge current using relations (12) and (13) we can obtain the current amounts in the beginning and in the end of the coil for various quantities of injection locations of partial discharge pulse where there are selected points for the injection of partial discharge pulse. To study the model, the partial discharge pulse similar to the data obtained from lab measurements was injected into the turns of 6 and 24 (K=6,24) of the simulated winding and the signal of the winding head which was a response to the PD pulse was measured. Fig. 11 shows the partial discharge pulse and Fig.12 shows the frequency spectrum of this signal.

![Image of partial discharge pulse and frequency spectrum](image)

**Fig. 10.** The measured and calculated magnitude of transfer function of turn 20 compared to the beginning of the winding

**Table 2.** The calculation and FEM results

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Calculated parameters by Common formulas</th>
<th>Identified parameters by FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_1) [(\mu)H]</td>
<td>(2.96 \times 10^{-7})</td>
<td>(3.13 \times 10^{-7})</td>
</tr>
<tr>
<td>(L_2) [(\mu)H]</td>
<td>(2.52 \times 10^{-7})</td>
<td>(2.58 \times 10^{-7})</td>
</tr>
<tr>
<td>(C_1) [(\mu)F]</td>
<td>(3.088 \times 10^{-10})</td>
<td>(3.244 \times 10^{-10})</td>
</tr>
<tr>
<td>(C_2) [(\mu)F]</td>
<td>(3.331 \times 10^{-10})</td>
<td>(3.638 \times 10^{-10})</td>
</tr>
<tr>
<td>(R_{\text{K}})</td>
<td>(0.01312)</td>
<td>(0.01442)</td>
</tr>
</tbody>
</table>

Also, in laboratory the PD pulse with the specifications of Fig. 11 is applied to 6 and 24 turns and is measured from the head of the winding. The partial discharge measurement circuit is as shown in Fig. 13.

These measured signals are in time domain according to Fig.14, and are transferred to frequency domain to be compared with the simulation results.

Fig. 15 shows the measured signals transferred to frequency domain.

Fig. 16 shows the output results obtained from simulation and laboratory. By comparing the obtained results in the figure, it is observed that the result of simulation is very close to the results obtained from laboratory. The small difference between the results shows the accuracy and good validity of this model.
To perform more studies on the results, the correlation coefficient was applied by the use of formula (25) for all the turns resulting from simulation and measurement depicted in Table 3.

\[ R(x, y) = \frac{\sum_i x_i y_i}{\sqrt{\sum_i x_i^2 \sum_i y_i^2}} \]  

(25)

The results of Table 3 show that the diametric coefficients (the correlation coefficients in the simulated turns with the measured turns of the same location) show the most dependence on each other. This shows that the simulation results enjoy good accuracy.

In general, these results show that the proposed model can properly study the partial discharge pulse propagation along the length of the winding.

5. Conclusion

The transformer winding was simulated by the use of multi-conductor transmission line in this paper. The efficiency of the multi-conductor transmission line model is greatly dependent on the MTL model parameters in the studies of transformer internal phenomena. With regard to inevitable approximations present in the calculation of the MTL model formula and also transformer generated tolerances along with the model intrinsic limitations, it is not practically possible to calculate the MTL model parameters with high accuracy in most cases. It was shown in this paper that it was possible to estimate the MTL parameters with high accuracy by the use of finite elements model (FEM) through Ansoft Maxwell software. The comparison of the results obtained through a test on a 20 kV winding and the results obtained from simulation of this winding is indicative of the accuracy and validity of this model.
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References


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