Optimal Location and Sizing of Shunt Capacitors in Distribution Systems by Considering Different Load Scenarios

Mohammadhosein Dideban†, Noradin Ghadimi*, Mohammad Bagher Ahmadi**
and Mohammad Karimi***

Abstract – In this work, Self-adaptive Differential Evolutionary (SaDE) algorithm is proposed to solve Optimal Location and Size of Capacitor (OLSC) problem in radial distribution networks. To obtain the SaDE algorithm, two improvements have been applied on control parameters of mutation and crossover operators. To expand the study, three load conditions have been considered, i.e., constant, varying and effective loads. Objective function is introduced for the load conditions. The annual cost is fitness of problem, in addition to this cost, CPU time, voltage profile, active power loss and total installed capacitor banks and their related costs have been used for comparisons. To confirm the ability of each improvements of SaDE, the improvements are studied both in separate and simultaneous conditions. To verify the effectiveness of the proposed algorithm, it is tested on IEEE 10-bus and 34-bus radial distribution networks and compared with other approaches.

Keywords: Annual cost, Differential evolutionary algorithm, Optimal capacitor allocation, Radial distribution networks

1. Introduction

Majority of loads in power systems use reactive power. In a power system, active power in several MWS is only generated by synchronous generators, while reactive power is produced not only by synchronous generator but also is injected by the other devices such as: Static VAr Compensator (SVC), Synchronous Condenser (SC), and Capacitor. Among these equipments, capacitor has the slowest and stepped speed response while installation and operating costs of capacitor are considerably lower than the other reactive power sources. Despite technical limits of capacitors, a capacitor could be a better option to generate reactive power at least for the economic advantages. The Optimal Location and Size of Capacitor (OSLC) problem has been solved by many techniques. In this paper, these techniques have been categorized in three classes; numerical and mathematical methods, heuristic and artificial intelligence techniques.

In [1-3], a set of numerical programming approaches have been proposed. To solve OLSC problem a computational method were suggested in [1], in first step of this method the candidate buses for capacitor installation, optimal size and proper type of capacitors (fixed or switching) are selected. In 2008, Khodr and et al. carried out similar work in [2]. Jabr has proposed a two stage technique to minimize OLSC problem in the presence of fixed and switching capacitors [3]. In first and second stages, OLSC problem is formulated as a conic program and a mixed integer linear program (MILP) based on minimizing the L1-norm, respectively.

The heuristic methods are in the other category of methods which have been suggested for solving OLSC problem in [4, 5]. The proposed technique in [4] uses the solution from the mathematical model after relaxing the integrality of the discrete variables to elect candidate buses for installation capacitor banks. Da Silva et al. have proposed a technique by nonlinear mixed integer optimization to solve OLSC problem. For this, sigmoid function was used to determine capacitor location and then the problem is formulated using the primal-dual interior point method [5]. In addition to heuristic methods, meta-heuristic approaches have been proposed in [6, 7]. Also, in [8], Memetic Algorithm (MA) was proposed to solve OLSC problem in large distribution networks. The MAs are population-based methods which can be taken as an extension of Genetic Algorithms (GAs).

The artificial intelligence is third category of approaches for solving OLSC problem. The most used artificial intelligences to solve OLSC problem are Evolutionary Algorithm (EA), Swarm Intelligence Algorithm (SIA), Neural Networks (NNs) and fuzzy sets. Also, in [9-11], GA or its improved branches were suggested to solve optimal capacitor placement. Main problem of GA is low convergence velocity, which authors of [12] have claimed that this problem is declined by methods based on the reduction of the search space of GAs or based on micro-
GAs. Abu-Elanien and Salama in [13] have suggested Discrete Wavelet Transform (DWT) integrated with a feed-forward artificial neural network (FFANN) for PQ improvement and solving OLSC problem, in simultaneous manner. Main problems of NNs are: network training is very difficult, obtained solutions’ accuracy is strongly depend on the size of training set, and finally predicting the future performance of the network (by popularity) is not possible.

Particle Swarm Optimization (PSO) is the most famous member of SIA. In [14], similar to [13], this work was performed using PSO. Hybrid PSO (HPSO) has been used to solve OLSC problem in unbalanced distribution in the presence of harmonics in [15]. This PSO obtained by combining PSO and radial distribution power flow algorithm. Main disadvantages of PSO are: high possibility of lying on local optimum point, especially in problems with large size and dimensions. Ant colony is another approach among SIA used by Chang for reconfiguration, capacitor placement and for loss reduction. Theoretical discussion of ant colony is difficult and is time consuming for convergence.

Fuzzy is free of problem structure and can be combined with other algorithms, then, in [16, 17], fuzzy set has been composed with ant colony, Immune Algorithm (IA) and GA, respectively. Creating membership function of fuzzy sets is difficult and in most cases is not viable.

In this paper, a novel algorithm is used to solve OLSC problem. The proposed SaDE algorithm is obtained by applying two improvements on original DE algorithm. Main goal of these improvements is self-adapting of two important control parameters of mutation and crossover operators. The fitness is a function of annual cost which is presented for two scenarios; constant and varying load conditions. The system load has been modeled in three patterns; constant, varying and effective patterns. In addition to objective function, in case studies, voltage profile and power loss and its related cost and total installed capacitor banks and its corresponding cost and CPU time have been used for comparison criteria. To illustrate effect of each improvement, the results of these improvements have been presented separately and compared with SaDE. Simulations have been implemented on IEEE 10-bus and 34-bus radial distribution networks.

2. Optimal Location and Sizing of Capacitor Problem

The optimal location and sizing of capacitor problem has been formulated with different goals. In majority studies, main target of capacitor installation is minimizing annual cost. In this paper, OLSC problem has been formulated as function of annual cost. In this study, to model different load conditions, three load patterns used; constant, varying and effective load.

2.1 Constant load

In this load pattern, it is assumed that load of system is constant. This condition is the simplest pattern. First term of objective function is capacity of installed capacitor banks multiplied by corresponding cost. The second term is total power loss of network multiplied by related cost.

\[
\text{Min} \left[ \sum_{i=1}^{NC} C_{\text{oper}} \times Q_i + C_{\text{loss}} \times \sum_{i=1}^{NB} P_{\text{loss}} \right] \tag{1}
\]

where, \(C_{\text{oper}}\) and \(C_{\text{loss}}\) are costs of power loss, in $/kW/year, and operation of each capacitor bank, in $/kVAR, respectively. \(Q_i\) and \(P_{\text{loss}}\) are capacity of capacitor bank, in kVAR, and the total active power loss of network, in kW, respectively. \(NC\) and \(NB\) are the total number of capacitors and bus, respectively.

2.2 Varying load

The second load pattern is varying load. In this load condition, the load of network changes in duration year. For this, several load levels and related durations are defined. The difference between objective function of constant and varying load pattern is the second term of objective function. In varying load, to apply duration of each load level, energy loss is inserted in objective function.

\[
\text{Min} \left[ \sum_{i=1}^{NC} C_{\text{oper}} \times Q_i + \sum_{k=1}^{NLL} \sum_{i=1}^{NB} C_{\text{loss}} \times (P_{\text{loss},T_i} \times T_k) \right] \tag{2}
\]

where, \(P_{\text{loss},T_i}\) and \(C_{\text{loss}}\) are power loss of any load level, in kW; and cost per energy loss, in $/kWh/year. \(T_k\) is the duration of \(k\)th load level. \(NLL\) is yearly total number of load levels.

2.3 Effective load

The load levels of varying load condition has an effective level which its value is calculated by Eq. (3) and applied in Eq. (1).

\[
S_{\text{eff}(i)} = \frac{\sum_{j=1}^{NLL} T_j \times S_j}{\sum_{j=1}^{NLL} T_j} \tag{3}
\]

where, \(T_i\) and \(S_j\) are duration of the \(i\)th load level, in hour, and the \(i\)th load level, in pu. Several constrains should be considered to solve OLSC which are visible in [18]

3. Differential Evolutionary Algorithm

The Differential Evolutionary (DE) algorithm was
proposed by Storn and Price in 1997 [19]. To start algorithm, first an initial population is generated and then to find global optimal point using mutation and crossover operators, the population are changed. In following quintet steps have been presented.

i) Initialization - To generate initial matrix, thee lower and upper limitations are selected. The difference of these limitations multiplied to a random value in range [0, 1] and then summed with upper boundary.

\[ u_{ik}^{G} = u_{ikmin} + \text{rand} \cdot [0,1] \times (u_{ikmax} - u_{ikmin}), \quad i \in [1, PN], \quad k \in [1, VN] \]  

(4)

In Eq.(9) \( u_{ikmin} \) and \( u_{ikmax} \) are lower and upper boundaries of the \( j \) component, respectively, which are selected based on the type of problem. \( \text{rand} \) is a random value between [0,1]. \( PN \) and \( VN \) are the number of population and variables, respectively.

ii) Mutation - After generation of initial population by mutation operator, vectors of population are changed and modified randomly. In this process, three random vectors, \( U_a, U_b \) and \( U_c \), are placed in Eq. (5),

\[ u_{ik}^{(G)} = u_{ik}^{(G)} + SF \cdot (U_{ik}^{(G)} - U_{ik}^{(G)}), \quad i = 1,...,PN \]  

(5)

where, \( a, b \) and \( c \) are randomly selected from the set \{1,...,PN\}, that \( a \neq b \neq c \neq i \), and also \( SF \) is a real and constant factor \([0,2]\).

iii) Crossover - All population has not recombination ability because corresponding fitness to them are far away from the global optimum, then crossover operator is applied on population. If crossover rate is more than random number in range [0, 1], vectors produced from mutation step are selected, otherwise, selection is performed from initial population. \( CR \) is real value in the range [0, 1].

\[ u_{ij}^{(G)} = \begin{cases} u_{ij}^{(G)} & \text{if } \rho_j \leq CR \text{ or } j = q \\ u_{ij}^{(G)} + SF \cdot (U_{ij}^{(G)} - U_{ij}^{(G)}) & \text{otherwise} \end{cases} \]  

(6)

where, \( \rho \) and \( q \) are chosen randomly from \([1,..,PN]\)

iv) Selection - Selection operator selects vectors corresponding to the best solution for next generation.

\[ u_{ij}^{(G+1)} = \begin{cases} u_{ij}^{(G)} & \text{if } f(U_{ij}^{G}) \leq f(U_{ij}^{G}) \\ u_{ij}^{(G)} + SF \cdot (U_{ij}^{(G)} - U_{ij}^{(G)}) & \text{otherwise} \end{cases} \]  

(7)

where, \( f(U_{ij}^{G}) \) and \( f(U_{ij}^{G}) \) are fitness corresponding to vectors of crossover and initialization steps, respectively.

v) Termination Criterion - This algorithm terminates if one of two termination criteria is satisfied; i.e., (i) an acceptable solution is obtained, this termination criterion is used when optimal solution of problem is given. (ii) The number of iteration reaches to the predetermined iteration number. In all optimization problem solution this criterion is used.

4. Self-adaptive Differential Evolutionary Algorithm

In many studies, better solutions have been extracted from original DE algorithm by applying improvements on simple DE. In general, applied improvements on original DE have two categories; adaptive approaches and structure change. Main problem of Evolutionary Algorithms (EAs) is proper value allocation for control parameters. The DE has three control parameters: \( SF, CR \) and population. In adaptive approach these parameters are selected dynamically and not by trial and error technique. The SaDE has two improvement steps. The SaDE has two improvement steps. These improvements are applied on two control parameters; i.e. \( SF \) and \( CR \). The capability and reliability of SaDE is confirmed by test on 21 test function [20].

4.1 SaDE-i: Self-adapting SF

In second step of original DE, \( SF \) is multiplied by the difference of two selected vectors. Value of \( SF \) is selected from range \([0, 2]\), randomly. The more the value of \( SF \) is lower (close to zero), the larger is the effect of the first selected vector, \( U_a \), and the lesser is the search space. But if \( SF \) was large (close to 2), search space was larger. If the search space is too large maybe the algorithm go away from the global optimum solution and if it is too small the mutation step is useless. The \( SF \) is defined in range \([0,2]\). While, in practical issue, to reach the optimal point it varies from 0.4 to 1[21]. Thus, in Eq. (8) a novel approach is proposed for \( SF \),

\[ SF_{i+1}^{G} = \begin{cases} SF_{i}^{G} + \text{rand}_i \cdot [0,1] \times SF_{i}^{max} & \text{if rand}_i < \tau_i \\ SF_{i}^{G} & \text{otherwise} \end{cases} \]  

(8)

where, \( SF_{i}^{max} \) and \( SF_{i}^{max} \) are lower and upper limits of \( SF \). To obtain optimal value, \( SF_{i}^{min} \) and \( SF_{i}^{max} \) are adjusted on 0.4 and 1.0, respectively. The selection criteria between a new value for \( SF \) and old value of \( SF \) are \( \tau_i \). If \( \tau_i \) was lower than a random value in range \([0,1]\), a new value is generated by first scenario of Eq. (8), otherwise value of \( SF \) is maintained fixed for next case without any changes.

4.2 SaDE-ii: Self-adapting CR

The \( CR \) is main control parameter. This parameter is selected in range \([0, 1]\) by operator. For the next step, \( CR \) determines which vectors of initial or mutation is selected.
In original DE, the CR is selected experimentally or by trial and error method. In SaDE, CR is generated by Eq. (9),

$$CR_{G+1}^{i} = \begin{cases} 
\text{rand}, & [0,1] \text{ if } \text{rand} < \tau_2 \\
CR^i_c, & \text{otherwise}
\end{cases} \quad (9)$$

The role of $\tau_2$ is same as role $\tau_1$ in SaDE-i.

### 4.3 Adjustment for OLSC problem

In this work, self-adaptive DE algorithm has been proposed to solve optimal capacitor placement. The algorithm has two improvement phases; in first and second phases, novel equations for SF and CR has been used, respectively.

The solution of OLSC problem using SaDE algorithm, at first the size of population, generation, buses, capacitor type, values for base voltage and power, consumed active and reactive powers and lines impedance are applied. By load flow program, voltage profile and power loss of test system have been extracted. Then SaDE algorithm is initialized by Eq. (4) and objective function (OF) is computed based on initial values. After, mutation and crossover operators are applied on initial population based on Eqs. (5-6). Load flow is recomputed and OF is calculated again by new values. The selection operator selects best solution between two obtained OFs (after initialization and crossover). In this work, termination criterion is ending the number of iteration, and then this algorithm is repeated until the maximum number of iteration is reached. Flowchart of OLSC problem solution by SaDE algorithm has been illustrated in Fig. 1.

### 5. Case Study

In this section, to confirm robustness of SaDE in solving OLSC problem, three load patterns are tested on two test systems. The constant load is applied on IEEE 10-bus, and both varying and effective loads are applied on IEEE 34-bus standard radial network. A backward-forward load flow approach used in this paper is same as in [22]. Cost per power loss and cost per energy loss are 168 $/kW/year and 0.06 $/kWh/year, respectively [23]. $\tau_1$ and $\tau_2$ are equal to 0.1 [20]. The capacity and related cost of capacitor banks have been presented in [23].

The SaDE has two improvement steps. To show the capability of each improvement steps, simulations are carried out for each step separately. Then, SaDE-i and SaDE-ii show first and second improvements (Eq.(7) and Eq.(8)), respectively. It should be noted again that SaDE is composed of SaDE-I and SaDE-ii. To compare different methods, seven parameters have been introduced which are: annual cost, in $, total installed capacitor bank, in kVAr, CPU time, in sec, power loss, in kW, and their related costs, in $, minimum voltage, in pu, and annual cost, in $.

#### 5.1 Constant load

The constant load is applied on IEEE 10-bus standard radial network. Topology of IEEE 10-bus has been illustrated in Fig. 2 [29]. Table 1 shows results of OSLC problem in IEEE 10-bus radial network with constant load.

<table>
<thead>
<tr>
<th>Meth.</th>
<th>CPU Time</th>
<th>Power Loss</th>
<th>Min.Volt.</th>
<th>Cost Loss+Cap. Annual</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>783.8</td>
<td>0.9375</td>
<td></td>
<td>0+131675=131675</td>
</tr>
<tr>
<td>2</td>
<td>681.28</td>
<td>0.9001</td>
<td></td>
<td>1865+114455=116320</td>
</tr>
<tr>
<td>3</td>
<td>675.43</td>
<td>0.9001</td>
<td></td>
<td>1932+113472=115404</td>
</tr>
<tr>
<td>4</td>
<td>675.37</td>
<td>0.9002</td>
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<tr>
<td>5</td>
<td>675.36</td>
<td>0.9002</td>
<td></td>
<td>1930+113460=115393</td>
</tr>
</tbody>
</table>

![Flowchart of OLSC problem solution by SaDE algorithm](image1.png)

![IEEE 10-bus distribution network](image2.png)
In this table, results of various SaDE algorithms have been compared with results of hybrid method which is created by compositing fuzzy and genetic algorithm [19]. Methods 1, 2, 3, 4 and 5 are without capacitor, hybrid, SaDE-ii and SaDE, respectively.

By considering results of Table 1, SaDE algorithm has the best solution. The CPU time for SaDE is the minimum among SaDE family and is less 6.1613 and 2.1529 sec respect to the related parameter of SaDE-i and SaDE-ii algorithms, respectively. The active power of SaDE is less than hybrid, SaDE-i, and SaDE-ii methods; the differences are 5.92, 0.07 and 0.01 kW, respectively. The minimum voltage of hybrid and SaDE-i approaches are close to each other and also less than minimum voltage of SaDE and SaDE-ii algorithms. The power loss cost of SaDE algorithm is less than hybrid, SaDE-i and SaDE-ii algorithms; the related amounts are 995, 12, and 2 in $i$, respectively. The annual cost of SaDE is equal to SaDE-ii, the values are 972 and 11 $i$ less than hybrid and SaDE-ii algorithms, respectively. Fig. 2 shows optimal location/size of installed capacitor banks in IEEE 10-bus radial network. In this figure, presented values for capacitor banks is in term of $kVAR$.

The presented optimal location/size of SaDE and SaDE-ii algorithms are same and in most cases similar to SaDE-i algorithm. The number of capacitor banks of hybrid method is less than the number of capacitor banks of each SaDE family.

### 5.2 Varying load

In practical cases, the network load is changing daily. To study varying load, three load levels are defined; i.e. 1st load level with 1.0 $pu$ load for duration 6780 $h$, 2nd load level with 0.8 $pu$ load for duration 6780 $h$ and 3rd load level with 0.5 $pu$ for 1000 $h$. The IEEE 34-bus radial distribution network is test case of varying load condition (see Fig. 3) [29]. The results of varying load simulation have been listed in Table 2. In all cases, the best solution has been bolded, while the worst are crossed with a line.

According to results of Tables 2, the minimum voltage of SaDE approach, in all load levels, are best solution. For power loss and its related cost and total installed capacitor banks and its related cost, SaDE-ii and SaDE-i algorithms give relatively better options, while annual cost of SaDE-ii technique is the worst solution. In first level, CPU time of SaDE algorithm is minimum amount among SaDE family and this value is less for 10.7442 and 5.7873 sec less than SaDE-i and SaDE-ii techniques, respectively. SaDE-i

![Fig. 3. Location and size of installed capacitors on 34-bus with Varying load](image)

### Table 2. Results of capacitor placement on 34-bus with varying load

<table>
<thead>
<tr>
<th>Method</th>
<th>Level</th>
<th>CPU Time</th>
<th>Power Loss</th>
<th>Min. Volt.</th>
<th>Cost($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W/O Cap</td>
<td>1</td>
<td>221.72</td>
<td>0.9492</td>
<td>13303</td>
<td>13303</td>
</tr>
<tr>
<td>Hybrid</td>
<td>2</td>
<td>139.16</td>
<td>0.9609</td>
<td>56443</td>
<td>56443</td>
</tr>
<tr>
<td>SaDE-i</td>
<td>3</td>
<td>52.855</td>
<td>0.9783</td>
<td>3171</td>
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<tbody>
<tr>
<td>W/O Cap</td>
<td>1</td>
<td>160.5</td>
<td>0.9486</td>
<td>9630</td>
<td>9630</td>
</tr>
<tr>
<td>Hybrid</td>
<td>2</td>
<td>101.18</td>
<td>0.9593</td>
<td>41041</td>
<td>41041</td>
</tr>
<tr>
<td>SaDE-i</td>
<td>3</td>
<td>56.439</td>
<td>0.9608</td>
<td>40550</td>
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<tr>
<td>W/O Cap</td>
<td>1</td>
<td>59.427</td>
<td>0.9753</td>
<td>2354</td>
<td>2354</td>
</tr>
<tr>
<td>Hybrid</td>
<td>2</td>
<td>55.825</td>
<td>0.9753</td>
<td>40513</td>
<td>40513</td>
</tr>
<tr>
<td>SaDE-i</td>
<td>3</td>
<td>55.905</td>
<td>0.9753</td>
<td>40576</td>
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algorithm presents best solution for capacitor banks cost. From viewpoint of annual cost SaDE-ii is the worst case among SaDE family. In first and second load levels, SaDE presents best solution. Proposed optimal location/size of installed capacitor banks by SaDE family and hybrid method have been presented in Fig. 3.

In all load levels, the number of capacitor banks proposed by SaDE-ii algorithm is more than other approaches. In first load level, the number of capacitor banks proposed by SaDE and SaDE-i is equal to each other, and is 1 and 5 capacitor banks less than hybrid and SaDE-ii algorithms, respectively. In 2nd load level, the number of installed capacitor banks of SaDE-ii algorithm is 3 banks more than related parameter of SaDE-i and SaDE techniques. In third load level, this deference is same for second load level.

5.3 Effective load

The varying load has an effective level which is calculated by Eq. (3). For the introduced three load levels, effective load level is equal to 0.78858 pu. Tables 3 and Fig. 4 show solution results of OLS problem for effective load and their optimal location/size of installed capacitor banks of in IEEE 34-bus radial distribution network, respectively. Methods 1, 2, 3 and 4 are hybrid, SaDE-i, SaDE-ii and SaDE approaches, respectively.

By considering results of Table 3, it is obvious that the power loss of SaDE-ii has the least value, and is 8.16, 0.389, and 0.356 kW less than hybrid, SaDE-i, as well as SaDE, respectively. The least CPU time of effective load is obtained by SaDE-i algorithm, CPU time of SaDE-i algorithm is 0.5348 and 1.6033 second less than thees of SaDE and SaDE algorithms, respectively.

The minimum voltage of SaDE-ii and SaDE are the most and least minimum voltages among SaDE family. The SaDE-ii algorithm presents optimal power loss cost being 1372, 66, and 60 $ less than hybrid, SaDE-i, and SaDE approaches, respectively. The total cost of SaDE algorithm is optimal value among four methods and is 1158, 107, and 303 $ less than annual cost of hybrid, SaDE-i, and SaDE-ii, respectively.

5.4 Comparison

In this part, to analyze performance of these three approaches to solve OLS problem, two criteria are used. First criterion is the number of best or worst solution for each technique among solutions. The criterion shows reliability of any approach, the more the number of best solution, the more is the reliability of methods. The number of best/worst for SaDE family has been illustrated in Fig. 4. Results of SaDE family are better than hybrid method; this fact confirms capability of SaDE family to solve OLS problem. Thus, in Fig. 5, the obtained results by hybrid technique have been ignored.

Focusing on Fig. 5 reveals that SaDE-ii, and SaDE-i algorithms have better solution. The SaDE is the best option among these four algorithms; this algorithm has the minimum value in worst solution, and the maximum value

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<tbody>
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<td>1</td>
<td>-</td>
<td>0.96015</td>
<td>17653+474=18127</td>
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</tr>
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<td>2</td>
<td>51.570</td>
<td>0.96131</td>
<td>16347+729=17076</td>
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<td>52.105</td>
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<td>4</td>
<td>53.173</td>
<td>0.96013</td>
<td>16341+627=16969</td>
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</table>

Table 3. Results of capacitor placement on 34-bus with effective load

Fig. 4. Compression of location/size of installed capacitor on 34-bus with effective load

Fig. 5. The number of best/worst solution of SaDE family

Fig. 6. Error percentage of minimum voltage
for best solutions. Among SaDE family, SaDE-i technique is the worst option.

The other criterion is error percentage of base value respect to computed value. The base value of minimum voltage and power loss are 1 pu and 2000 kW, respectively. Fig. 5 shows error percentage of minimum voltage of constant and varying as well as effective loads.

Regarding results of Fig. 5, for minimum voltages, SaDE algorithm reaches better solution in most cases; this algorithm except effective load level has optimal value. The SaDE-ii algorithm has better solution among four approaches only in constant load and effective load level. In this case, SaDE-i has the worst solution among SaDE family and only in 3rd level of varying load presents optimal value which is equal to related parameter of SaDE algorithm. In constant load, minimum voltage error percentage of SaDE-ii is equal to corresponding parameter of SaDE which both is 0.011 and 0.013 less than hybrid and SaDE-i algorithms, respectively. In first level of varying load, the error percentage of SaDE is 0.159, 0.004 and 0.029 less than related values of hybrid, SaDE-i, SaDE techniques, respectively. In level 2, these reduction values are 0.159, 0.006 and 0.028. In third load level, minimum voltage error percentage of SaDE-i and SaDE algorithms is equal and is 0.037 and 0.064 less than hybrid and SaDE-ii techniques, respectively. Finally, in effective load level, error percentage of minimum voltage of SaDE-ii is 0.127, 0.011, and 0.039 less than error percentage of hybrid, SaDE-i, SaDE approaches, respectively.

After computation of minimum voltage error percentage, error percentage of power loss is computed by the approach which was used for minimum voltage. The base value of power is 2 MW. Fig. 6 shows error percentage of power loss for three load conditions.

It should be mentioned that, in this case, the more the power loss error percentage, the better is the solution. Among five cases, the SaDE-ii algorithm in most cases presents better solution. In error percentage of power loss similar to error percentage of minimum voltage, SaDE is the worst option among SaDE family with only one best solution.

The error percentage of SaDE, in constant load, is 0.292, 0.0035, and 0.0005 more than hybrid, SaDE-i, and SaDE-ii algorithms, respectively. In first level of varying load, SaDE-ii approach presents the best value which its value is 0.0835, 0.0780 and 0.059 more than hybrid, SaDE-i, and SaDE algorithms, respectively. These increments in level 2 are 0.0658, 0.0083 and 0.0078. The SaDE-I algorithm in 3rd level of varying and effective loads has best value. In 3rd level of varying load, the difference between error percentage of SaDE-ii and three other algorithms; i.e. hybrid, SaDE-ii and SaDE techniques are 0.0024, 0.0002 and 0.0036, respectively. Finally, in effective load level, SaDE-ii algorithm has maximum power loss error percentage and its value is 0.4083, 0.0195 and 0.0178 more than hybrid, SaDE-i, SaDE algorithms, respectively.

5.5 Discussion

In this study, to solve optimal capacitor allocation in radial distribution network a novel algorithm based on simple DE algorithm has been proposed. The proposed algorithm, SaDE, has two improvement steps: self-adapting $SF$ and CR called SaDE-i and SaDE-ii, respectively. From results of simulation and comparison, followings have been extracted:

**Remark i)** In addition to capacity of installed capacitor banks, dispatch manner of capacitor banks has considerably effects on cost. This fact has been extracted by comparing cost of capacitor among SaDE family, in level 2 of varying load between SaDE-i and hybrid, in level 3 of varying load between hybrid and SaDE-ii. Thus less installed capacitor always does not result in lower cost.

**Remark ii)** The numbers of buses have impact on CPU time more than total demand of network. In varying load, demand of 1st load level is twice of 3rd load level demand while CPU time changes only about 20%. This fact could be derived by comparing between 10-bus and 34-bus radial networks. The number of buses are one of initial matrix dimension, then the more the number of buses, the lower is convergence velocity.

**Remark iii)** Form the view point of voltage profile improvement, SaDE-ii is better than SaDE-i algorithm. The SaDE-i has the worst solution in SaDE family. This fact confirms that among control parameters of DE algorithm; crossover rate has the maximum impact on voltage profile. Then self-adapting CR helps to extract better solution from algorithm respect to adjust a constant value for CR in original DE. The better results of SaDE-ii is compared to SaDE illustrates that self-adapting $SF$ does not affect remarkably on voltage profile.

**Remark iv)** The capability of SaDE-i algorithm is confirmed in less installed capacitor banks presented by this algorithm. In constant load, total installed capacitor banks of SaDE family is equal, in effective load this parameter of SaDE and SaDE-i algorithms are 300 kVar less than SaDE-ii algorithm. Unlike SaDE-i, SaDE-ii in the most cases has largest installed capacity; the total installed capacitor banks in second and third level of varying load and effective load confirm this extraction. Thus, $SF$
improvement reduces installed capacitor banks more than CR improvement.

6. Conclusion

In this work, an improved DE algorithm, named SaDE, was used to solve OLSC problem. The proposed algorithm was obtained by self-adapting control parameter of mutation and crossover operators; i.e. CR and SF, respectively. Three load conditions, constant and varying as well as effective, were tested on 10-bus and 34-bus radial distribution networks. Fitness is function of annual cost, in addition to cost, six other parameters used for comparison were installed capacitor banks and related cost, CPU time, minimum voltage, active power loss and related cost. To compare results, the number of best/worst solutions and error percentage of minimum voltage and power loss were used.

From simulation results of case studies it can be claimed that: in general, self-adapting CR is more effective than self-adapting SF; latter has better solution for the number of installed capacitor banks and related cost. Sum of these two improvements has better overlapping and give an optimal solution. Cost of total installed capacitor banks is less than 10% of annual cost, remaining cost (90%) is power loss cost. Therefore, it is better to focus on power loss reduction. Load decline has less impact on the speed of algorithm running, and decreasing the load amount only 20% decrease the speed of algorithm running. In addition to capacitor banks amount, the number of installed capacitor banks also have remarkable impact on cost.

References

Optimal Location and Sizing of Shunt Capacitors in Distribution Systems by Considering Different Load Scenarios


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