Characteristic Analysis of an Traveling Wave Ultrasonic Motor using a Cylindrical Dynamic Contact Model

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Abstract – The traveling wave ultra-sonic motor (TWUSM) is operated through the frictional force between the rotor and the stator. Hence, the contact mechanism should be analyzed to estimate the motor performance. However, the nonlinearity of the contact mechanism of the TWUSM makes it difficult to propose a proper contact model and a characteristic analysis method. To address these problems, a novel contact model is proposed and be termed the cylindrical dynamic contact model (CDCM) in this research. An estimation method of the motor performance is proposed using the CDCM, an analytical method, and a numerical method. The feasibility and usefulness of the proposed characteristic analysis are verified through experimental data.

Keywords: Actuator, Contact, Energy conversion, Finite element method, Nonlinear, Traveling wave, Ultrasonic motor.

1. Introduction

The nonlinearity of the contact mechanism of the TWUSM has been made it difficult to propose an effective characteristic analysis method [1-6]. Hence, many researchers have investigated the contact mechanisms of TWUSMs over the past few decades. Numerous types of characteristic analysis methods using diverse kinds of contact models have been proposed for the TWUSM by many researchers [2, 7-13]. However, some of the proposed methods have to use the unreasonable values of material coefficients or inconsistent material coefficients to fit the calculated data to the simulated results. In some methods, the governing equations are complex to be solved. Some methods can analyze the partial characteristics of the motor performance. To address these conventional problems, a novel contact model for the TWUSM, which referred to CDCM, is proposed in this research. Furthermore, a characteristic analysis method using the proposed CDCM, an analytical method, and a numerical method for the TWUSM is proposed in this paper. The proposed method for the TWUSM is verified by experimental data with a prototype of the TWUSM.

2. Working Principle of the TWUSM

Fig. 1 illustrates the working principle of the TWUSM.

3. Characteristic Analysis Method using the CDCM

3.1 CDCM

As shown in Fig. 2, we assumed the stator and the rotor of the TWUSM as cylindrical bodies with the radius of $R_1$ and $R_2$ respectively. To reflect the dynamic contact condition, the CDCM is proposed as illustrated in Fig. 3,
describing the nonlinear deformation at the area of contact. It is assumed that both rolling and sliding occur between the stator and the rotor while they are in contact.

3.2 Analysis of TWUSM using the FEM

The first step for the characteristic analysis of the TWUSM is the calculation of the operating frequency \( f \) (Hz) and the displacement amplitude \( A \) (m) of the TWUSM by means of 3D-FEM. We developed the 3D-FEM software and an auto-mode search program. The operating frequency can be calculated through an impedance analysis using FEM and (1), in which \( Z(\omega) \) is the impedance at the excitation frequency \( \omega \) (rad/sec), \( \Phi(\omega) \) (V) is the electric potential on the electrode, and \( Q_0 \) (C) is the electric charge.

\[
Z(\omega) = \frac{\Phi(\omega)}{j\omega Q_0}
\]

(1)

3.3 Formula for the motion of a stator

Based on Kirchhoff’s plate theory, the neutral plane of the stator can be expressed in terms of an ideal traveling wave as (2) while assuming that the stator is not affected by the contact force [1-3, 11]. Here, \( r \) is the position in the radial direction, \( R(r) \) is the amplitude at radius \( r \), and \( n \) is the number of nodal lines.

\[
w(r, \phi, t) = R(r) \cos \left( n\phi - \omega t \right)
\]

(2)

The three-dimensional problem can be replaced by a two-dimensional problem, as expressed by (3).

\[
w(\hat{x}, t) = A \cos \left( \frac{2\pi \hat{x}}{\lambda} - \omega t \right)
\]

(3)

which is subject to

\[
\hat{x} = x + v_w t
\]

(7)

\[
v_w = \frac{\lambda \omega}{2\pi}
\]

(8)

The moving reference follows the wave crest. Hence, (3) can be formulated into the time-independent Eq. (9) by substituting (7) and (8) into (3). The normal displacement of the neutral plane of the stator in the moving reference can be derived as (10).

\[
w(\hat{x}, t) = w(x + v_w t, t) = A \cos \left( \frac{2\pi \hat{x}}{\lambda} - \omega t \right)
\]

(9)

\[
w(x, t) = A \cos \left( \frac{2\pi x}{\lambda} \right)
\]

(10)

3.4 Analysis of the contact length

When two cylinders, which are the stator and the rotor, come into contact with each other, as shown in Fig. 2, the half of the contact length \( L \) can be calculated by (11), where \( m_1 \) and \( m_2 \) are the material coefficients of the stator and the rotor, respectively; \( B \) is the shape coefficient of the cylinder; \( F_{e,\text{avg}} \) (N/m) is the normal force per unit length in the radial direction at the unit wave; \( E_1 \) (Pa) and \( E_2 \) (Pa) are the equivalent elasticity modulus for the stator and the rotor, respectively; \( v_1 \) and \( v_2 \) are the Poisson’s ratios of the stator and the rotor, respectively; \( F_N \) (N) is the normal force applied to the TWUSM for the frictional force; and \( b \) is the contact width in the radial direction.

\[
L = \frac{m_1}{m_2} \cdot \frac{\lambda}{n} - \frac{B}{2}\left( \frac{d_{\text{out}} + d_{\text{in}}}{4} \right)
\]

(11)
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\[ L = \sqrt{\frac{2m_1 + m_2}{\pi B} F_{N\text{-unit}}} \]  

(11)

which is subject to

\[ m_1 = \frac{1 - \nu_1^2}{E_1} \]  

(12)

\[ m_2 = \frac{1 - \nu_2^2}{E_2} \]  

(13)

\[ F_{N\text{-unit}} = \frac{F_N}{nb} \]  

(14)

The equivalent elasticity moduli \( E_1 \) and \( E_2 \) should be calculated while taking the composite material of the TWUSM into account. The stator consists of piezoelectric ceramic material with a metal medium. The rotor is composed of the contact material and a metal medium. The equivalent elasticity modulus of the composite material can be derived by (15) when two materials are stacked in the thickness direction; their elasticity moduli are \( E_a \) and \( E_b \) with thicknesses of \( t_a \) and \( t_b \), respectively.

\[ E_{\text{tot}} = \frac{E_a E_b (t_a + t_b)}{t_a E_b + t_b E_a} \]  

(15)

The shape coefficient of cylinder \( B \) can be calculated by (16) [15]. \( R_1 \) can be calculated using the displacement of the stator by (17). \( R_2 \) is an infinite value because the rotor is a plate.

\[ B = \frac{1}{2} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \]  

(16)

\[ \frac{1}{R_1} = \frac{\partial^2 w(x)}{\partial x^2} \bigg|_{x=0} = \left( \frac{2\pi}{\lambda} \right)^2, \quad w(x) \bigg|_{x=0} = \left( \frac{2\pi}{\lambda} \right)^2 \cdot A \]  

(17)

3.5 Analysis of the normal pressure distribution

Because the normal pressure distribution is proportional to (10), which is the normal displacement of the neutral plane of the stator, the normal pressure distribution \( p(x) \) (N/m\(^2\)) on the contacting surface can be derived by (18), where \( X \) (N/m\(^2\)) is the amplitude of the normal pressure distribution before the deformation of the stator and where \( C \) is an unknown constant. The unknown constant \( C \) accounts for the boundary conditions, which state that \( p(x) \) should be equal to zero when \( x=-L \) and \( x=L \). Consequently, \( C \) can be found by (19). The unknown function, \( X \), can be found using condition (20). Eq. (21) can be induced from the condition under which the normal pressure distribution is symmetric about \( x=0 \) and by substituting (18) and (19) into (20). From (21), the amplitude of the normal pressure distribution \( X \) is derived by (22).

\[ p(x) = X \left[ \cos \left( \frac{2\pi x}{\lambda} \right) + C \right] \]  

(18)

\[ C = -\cos \left( \frac{2\pi L}{\lambda} \right) \]  

(19)

\[ F_{N\text{-unit}} = \int_{-L}^{L} p(x) \, dx \]  

(20)

\[ F_{N\text{-unit}} = 2X \int_{0}^{L} \left[ \cos \left( \frac{2\pi x}{\lambda} \right) - \cos \left( \frac{2\pi L}{\lambda} \right) \right] \, dx \]  

(21)

\[ X = \frac{\pi F_{N\text{-unit}}}{\lambda \sin \left( \frac{2\pi L}{\lambda} \right) - \frac{2\pi L}{\lambda} \cos \left( \frac{2\pi L}{\lambda} \right)} \]  

(22)

Finally, the governing equation for the analysis of the normal pressure distribution as a function of the normal force applied to a TWUSM, \( F_N \), is derived by (23). The maximum pressure \( p_{\text{max}} \) can be calculated by \( p(0) \).

\[ p(x) = \frac{\pi F_N}{nb \lambda \sin \left( \frac{2\pi L}{\lambda} \right) - \frac{2\pi L}{\lambda} \cos \left( \frac{2\pi L}{\lambda} \right)} \]  

(23)

3.6 Analysis of the stress and the strain

In the case of dynamic contact, the stress can be expressed as normal stress and tangential stress [14, 15]. The normal stress, \( \sigma_n(x,z) \), which is induced by normal force can be derived by (24). The tangential stress, \( \sigma_z(x,z) \), which is generated by the tangential frictional force can be expressed by (25).

\[ \sigma_n(x,z) = -\frac{Z}{\pi} \left[ L \beta(x,z) - \alpha(x,z) \right] p_{\text{max}} \]  

(24)

\[ \sigma_z(x,z) = -\frac{1}{\pi} z^2 \alpha(x,z) f_{\text{max}} \]  

(25)

which are subject to

\[ \alpha(x,z) = \frac{\pi}{k_1} \left[ 1 - \frac{k_3}{\sqrt{k_1}} \right] \]  

\[ \beta(x,z) = \frac{\pi}{k_1} \left[ 1 + \frac{k_3}{\sqrt{k_1}} \right] \]  

(26)

(27)

\[ k_1 = (L+x)^2 + z^2 \]  

(28)

\[ k_2 = (L-x)^2 + z^2 \]  

(29)
where \( \mu \) is the dynamic frictional coefficient.

The stress for the dynamic contact can be expressed by (31). From the Hook’s law, Eq. (32) is derived. The strain, \( \varepsilon \), can be induced by applying the infinitesimal thickness in the \( z \) direction, \( \Delta t_{sator} \), as (33).

\[
\sigma_z(x,z) = \sigma_z(x,z) + \sigma_z(x,z) + \sigma_z(x,z) + \sigma_z(x,z) + \sigma_z(x,z) + \sigma_z(x,z) \tag{31}
\]

\[
\varepsilon_z(x,z) = \frac{\delta_z(x,z)}{\Delta t_{sator}} \tag{32}
\]

\[
\delta_{tot}(x,z) = \sum_{z=0}^{\infty} \delta_z(x,z) \tag{33}
\]

Hence, the deformation at a segmented element, \( \delta(z,x) \), can be determined by (34). The total deformation in the \( z \) direction at \( x \) position, \( \delta_{tot}(x) \), is derived by (35).

\[
\delta_z(x,z) = \frac{\delta_z(x,z)}{E_z} \tag{34}
\]

\[
\delta_{tot}(x) = \sum_{z=0}^{\infty} \delta_z(x,z) \tag{35}
\]

### 3.7 Analysis of the displacement and the velocity distribution of the stator taking the deformation into consideration

The displacement amplitude of the stator decreased nonlinearly with the deformation caused by the applied normal force and the dynamic contact condition. The deformed normal displacement equation of the stator will be an asymmetric non-linear function due to the dynamic contact condition. Hence, an approximate method is proposed in this research to describe the nonlinear traveling wave of a deformed stator. The normal displacement of the neutral plane of a deformed stator in the plus \( x \) contact area, \( w_{final\_p}(x) \), and in the minus \( x \) contact area, \( w_{final\_m}(x) \), in the moving reference can be derived by (36) and (38), respectively. The traveling wave of the neutral plane of a deformed vibrator in the plus \( x \) contact area, \( w_{final\_p}(\hat{x},t) \), and in the minus \( x \) contact area, \( w_{final\_m}(\hat{x},t) \), in the fixed reference can be induced by (37) and (39), respectively. The total number of unknown constants of (36-39) is eight, in which \( \omega \), and \( \omega_m \) can be found as the normal displacement when \( x \) equal 0. The remaining six unknown constants can be calculated by solving a 3x3 inverse matrix using the three \( x \) position and the associated normal displacement values for each plus and minus contact area.

\[
w_{final\_p}(x) = w(x) - \delta_{tot\_z}(x) \tag{36}
\]

\[
w_{final\_m}(x) = w(x) - \delta_{tot\_z}(x) \tag{37}
\]

\[
w_{final\_p}(\hat{x},t) = (\alpha_p x^3 + \beta_p x^2 + \gamma_p x + \zeta_p) \cos(k\hat{x} - \omega t) \tag{38}
\]

\[
w_{final\_m}(\hat{x},t) = (\alpha_m x^3 + \beta_m x^2 + \gamma_m x + \zeta_m) \cos(k\hat{x} - \omega t) \tag{39}
\]

The horizontal displacement at the contacting surface of the stator can be derived by applying Kirchhoff’s plate theory to (36-39). The horizontal displacement for the plus \( x \) area in the fixed reference, \( w_{final\_p}(\hat{x},t) \), can be induced by (40), where \( a \) is the distance from the neutral plane to the contact surface.

\[
u_{s\_p}(\hat{x},t) = \frac{dw_{final\_p}(\hat{x},t)}{dx} = -a[\omega(3\alpha_p x^3 + 2\beta_p x + \gamma_p) \sin(k\hat{x} - \omega t)] \tag{40}
\]

The horizontal velocity of the stator at the contact surface in the fixed reference, \( v_{s\_p}(\hat{x},t) \), can be derived by (41) for the plus \( x \) area.

\[
v_{s\_p}(\hat{x},t) = \frac{du_{s\_p}(\hat{x},t)}{dt} = -a[\dot{\omega}(3\alpha_p x^3 + 2\beta_p x + \gamma_p) \sin(k\hat{x} - \omega t)] \tag{41}
\]

The horizontal velocity of the stator at the contact surface in the moving reference, \( v_{s\_p}(\hat{x},t) \), can be induced by (42) for the plus \( x \) area by applying (7) and (8) to (41).

\[
v_{s\_p}(x) = \frac{dx_{final\_p}(x)}{dt} = -a[\dot{\omega}(3\alpha_p x^3 + 2\beta_p x + \gamma_p) \sin(kx)] \tag{42}
\]

The governing equations for the minus \( x \) area can be suggested in the same way as (40-42).

### 3.8 Analysis of the normal pressure distribution considering the deformation of a stator

The normal pressure distribution induced by a nonlinear displacement function which considers the deformation of
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the stator at the plus x contact area of \( p_{\text{final}_p} \) (N/m²) can be determined by (43), where \( X_{\text{final}_p} \) is the amplitude of the final normal pressure distribution and \( C_{\text{final}_p} \) is an unknown constant at the plus x contact area. For the minus x area, the normal pressure displacement can be derived in the same way as (43). The unknown constants can be solved by (44) and (45).

\[
\begin{align*}
 p_{\text{final}_p}(x) & = X_{\text{final}_p} \left[ w_{\text{final}_p}(x) + C_{\text{final}_p} \right] \\
 & = X_{\text{final}_p} \left[ (\alpha_p x^3 + \beta_p x^2 + \gamma_p x + \zeta_p) \cos(kx) + C_{\text{final}_p} \right] \\
 C_{\text{final}_p} & = -w_{\text{final}_p}(L) \\
 & = -\left( \alpha_p L^3 + \beta_p L^2 + \gamma_p L + \zeta_p \right) \cos(kL) \\
 C_{\text{final}_m} & = -w_{\text{final}_m}(-L) \\
 & = -\left( \alpha_p L^3 + \beta_p L^2 + \gamma_p L + \zeta_p \right) \cos(kL)
\end{align*}
\]

(43)

The unknown functions \( X_{\text{final}_p} \) and \( X_{\text{final}_m} \) can be induced by conditions (46) and (47).

\[
\begin{align*}
 F_{N_{\text{unit}_p}} & = \int_0^L p_{\text{final}_p}(x) \, dx \\
 F_{N_{\text{unit}_m}} & = \int_0^L p_{\text{final}_m}(x) \, dx
\end{align*}
\]

(46) and (47)

The normal force per unit length in the radial direction at unit wave, \( F_{N_{\text{unit}}_p} \), is constituted with the force of the plus x contact area \( F_{N_{\text{unit}_p}} \) and that of the minus x contact area \( F_{N_{\text{unit}_m}} \), as expressed by (48) and (49), respectively.

\[
\begin{align*}
 F_{N_{\text{unit}_p}} & = \frac{\sum_{x=0}^{L} w_{\text{final}_p}(x)}{\sum_{x=0}^{L} w_{\text{final}_m}(x) + \sum_{x=0}^{L} w_{\text{final}_p}(x)} \times F_{N_{\text{unit}}} \\
 F_{N_{\text{unit}_m}} & = \frac{\sum_{x=0}^{L} w_{\text{final}_m}(x)}{\sum_{x=0}^{L} w_{\text{final}_m}(x) + \sum_{x=0}^{L} w_{\text{final}_p}(x)} \times F_{N_{\text{unit}}}
\end{align*}
\]

(48) and (49)

For the plus x contact area, the unknown function \( X_{\text{final}_p} \) can be determined using (43-49), as follows:

\[
\begin{align*}
 F_{N_{\text{unit}_p}} & = \int_0^L p_{\text{final}_p}(x) \, dx \\
 & = X_{\text{final}_p} \int_0^L \left[ (\alpha_p x^3 + \beta_p x^2 + \gamma_p x + \zeta_p) \cos(kx) + C_{\text{final}_p} \right] \, dx \\
 & = X_{\text{final}_p} \left[ F_{\text{int}_p} \right] \left[ F_{\text{int}_p} \right] \\
 & \quad \times F_{N_{\text{unit}_p}}
\end{align*}
\]

Eq. (50) can be modified to (51). For the minus contact area, (52) can be derived from (50).

\[
\begin{align*}
 X_{\text{final}_p} & = \frac{F_{N_{\text{unit}_p}}}{\left[ F_{\text{int}_p} \right]} \\
 X_{\text{final}_m} & = \frac{F_{N_{\text{unit}_m}}}{\left[ F_{\text{int}_m} \right]} \\
 & = \frac{F_{N_{\text{unit}_m}}}{\left[ F_{\text{int}_m} \right]
\end{align*}
\]

(51) and (52)

where the integration part can be expressed by:

\[
\begin{align*}
 F_{\text{int}_p} & = \left[ (\alpha_p k^3 x^3 \sin(kx) + 3\alpha_p k^3 x^2 \cos(kx) \right. \\
 & \quad - 6\alpha_p \cos(kx) - 6\alpha_p kx \sin(kx) \\
 & \quad + \beta_p k^3 x^2 \sin(kx) + 2\beta_p k^4 x \cos(kx) \\
 & \quad - 2\beta_p k^3 \sin(kx) + \gamma_p k^3 x^2 \cos(kx) + \gamma_p k^2 \cos(kx) \\
 & \quad + \zeta_p k^3 \sin(kx) + C_{\text{final}_p} k^4 \right] \\
 F_{\text{int}_m} & = \left[ (\alpha_p k^3 x^3 \sin(kx) + 3\alpha_p k^3 x^2 \cos(kx) \right. \\
 & \quad - 6\alpha_p \cos(kx) - 6\alpha_p kx \sin(kx) \\
 & \quad + \beta_p k^3 x^2 \sin(kx) + 2\beta_p k^4 x \cos(kx) \\
 & \quad - 2\beta_p k^3 \sin(kx) + \gamma_p k^3 x^2 \cos(kx) + \gamma_p k^2 \cos(kx) \\
 & \quad + \zeta_p k^3 \sin(kx) + C_{\text{final}_p} k^4 \right]
\end{align*}
\]

Finally, the normal pressure distribution taking into account the nonlinearly deformed stator is proposed in this research as (54) and (55).

\[
\begin{align*}
 p_{\text{final}_p}(x) & = \frac{F_{N_{\text{unit}_p}}}{\left[ F_{\text{int}_p} \right]} \\
 p_{\text{final}_m}(x) & = \frac{F_{N_{\text{unit}_m}}}{\left[ F_{\text{int}_m} \right]}
\end{align*}
\]

(54) and (55)

3.9 Analysis of the motor performance

The horizontal velocity distribution of a deformed stator considering dynamic contact has an asymmetric shape, as shown in Fig. 4. Hence, the slip distribution between the rotor and the stator can be described by Fig. 5.

From Fig. 6, the frictional force acting on the surface of the stator at unit wave, \( F_{p_{fric}} \) (N/m), can be induced by (56), where \( N \) (rpm) denotes the revolutions per minute of the rotor.

\[
\begin{align*}
 F_{p_{fric}} & = \frac{F_{N_{\text{unit}_p}}}{\left[ F_{\text{int}_p} \right]} \\
 & = \frac{F_{N_{\text{unit}_m}}}{\left[ F_{\text{int}_m} \right]}
\end{align*}
\]

(56)

Fig. 4. Distribution of the horizontal velocity at the stator and the rotor considering the nonlinear deformation induced by dynamic contact.
The total frictional force acting on the surface of the stator, $F_{\text{fric \_stator}}$ (N), can be derived by (59), which is subject to (57) and (58).

$$F_{\text{fric \_stator}} = -\mu b \int_{-L}^{L} \text{sign} \left[ v_x(x) - \frac{2\pi N}{60} r_{\text{eff}} \right] p_{\text{final \_stator}}(x) dx$$  \hspace{1cm} (59)

The total load force acting on the rotor, $F_{L\_tot}$ (N) can be expressed by $nbF_L$. It has the same meaning as the total friction force of the rotor, $F_{\text{fric \_rotor\_tot}}$ (N), as induced by (60), which also can be used for the calculation of the thrust force of the linear TWUSM.

$$F_{L\_tot} = F_{\text{fric \_rotor\_tot}} = -\mu b \int_{-L}^{L} \text{sign} \left[ v_x(x) - \frac{2\pi N}{60} r_{\text{eff}} \right] p_{\text{final \_rotor}}(x) dx$$  \hspace{1cm} (60)

The torque $M_f$ (Nm) and the speed $N$ (rpm) of the ring-type TWUSM can be analyzed by (61), which is subject to (57) and (58).
4. Verification of Analysis Method

As shown in Fig. 7, the stator is prototyped and its impedance is measured and calculated. As displayed in Fig. 7, the calculated data are in good agreement with the experimental results. Hence, it is verified that the 3D-FEM used in this research is correct.

The speed-torque is analyzed using the proposed characteristic analysis method derived from CDCM. These results are then compared to the experimental data, as illustrated in Fig. 8. The analysis results are fitted well with the experimental data. These results prove that the proposed characteristic analysis method is correct. It takes about 1-2 seconds for the analysis using the proposed characteristic analysis method. Hence, the usefulness of the proposed analysis method is validated. As displayed in Fig. 9, the stress distribution at the contact area of the stator shows an asymmetric and nonlinear shape due to the consideration of dynamic contact. The material coefficients used for the characteristic analysis are included in APPENDIX and are shown in Table 1.

![Fig. 9. Calculated stress distribution at the stator using the CDCM.](image)

5. Conclusion

The contact mechanism of the TWUSM is a complex nonlinear problem, which makes it difficult to propose a useful contact model and a feasible motor performance analysis method. Hence, it is noteworthy from the fact that a correct and useful characteristic analysis method using CDCM is proposed in this research.

### Appendix

1. PZT (z-axis poling):

   Mechanical stiffness matrix for the constant electric field $E_z$, $e^E$ (N/m²)
   
   \[
   \begin{bmatrix}
   13.25 & 6.94 & 6.46 & 0.0 & 0.0 & 0.0 \\
   6.94 & 13.25 & 6.46 & 0.0 & 0.0 & 0.0 \\
   6.46 & 6.46 & 10.52 & 0.0 & 0.0 & 0.0 \\
   0.0 & 0.0 & 0.0 & 2.68 & 0.0 & 0.0 \\
   0.0 & 0.0 & 0.0 & 2.68 & 0.0 & 0.0 \\
   0.0 & 0.0 & 0.0 & 0.0 & 3.16 & 0.0
   \end{bmatrix} \cdot 10^{10}
   
   Piezoelectric coefficient; $e$ (C/m²)
   
   \[
   \begin{bmatrix}
   0.0 & 0.0 & 0.0 & 0.0 & 12.82 & 0.0 \\
   0.0 & 0.0 & 0.0 & 12.82 & 0.0 & 0.0 \\
   -6.61 & -6.61 & 13.5 & 0.0 & 0.0 & 0.0
   \end{bmatrix}
   
   Permittivity for the constant mechanical strain $S$; $S$ (F/m²)
   
   \[
   \begin{bmatrix}
   7.32 & 0.0 & 0.0 \\
   0.0 & 7.32 & 0.0 \\
   0.0 & 0.0 & 6.28
   \end{bmatrix} \cdot 10^{-9}
   
   Density; $\rho$ (kg/m³) = 7500
   Permeability; $Q$ = 900

2. Elastic body of the stator (Phosphor-bronze):

   Mechanical stiffness matrix; $c_m$ (N/m²)
   
   \[
   \begin{bmatrix}
   179.75 & 96.79 & 96.79 & 0.0 & 0.0 & 0.0 \\
   96.79 & 179.75 & 96.79 & 0.0 & 0.0 & 0.0 \\
   96.79 & 96.79 & 179.75 & 0.0 & 0.0 & 0.0 \\
   0.0 & 0.0 & 0.0 & 41.481 & 0.0 & 0.0 \\
   0.0 & 0.0 & 0.0 & 41.481 & 0.0 & 0.0 \\
   0.0 & 0.0 & 0.0 & 0.0 & 41.481 & 0.0
   \end{bmatrix} \cdot 10^9
   
   Density; $\rho_m$ (kg/m³) = 8780
   Mechanical quality factor; $Q_m$ = 3000
References


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